Name: _____

**Directions:** Solve the following problems. Give supporting work/justification where appropriate.

1. **[2 points]** Find the coefficient of $x^5$ in $(2-x)^8$. (It is small enough that you will be able to do the computation by hand.)

2. **[2 points]** Use the Binomial Theorem to find a simple formula for $\sum_{k=0}^{n} \binom{n}{k} 8^k (-5)^{n-k}$. Your answer may involve factorials and/or binomial coefficients.

3. **[3 points]** Let $n \in \mathbb{N}$. Prove that if $n$ is odd and $\binom{n}{2}$ is even, then $n \equiv 1 \pmod 4$.

4. **[3 parts, 1 point each]** Let $A$ be a set of size $2n$. Answer each question with a simple formula in terms of $n$; your formula may involve factorials and/or binomial coefficients. No justification required.

   (a) How many subsets of $A$ are there?

   (b) How many subsets of $A$ contain exactly half the elements in $A$?

   (c) How many subsets of $A$ contain fewer than half the elements in $A$?