Name: ____________________

Directions: Solve the following problems. Give supporting work/justification where appropriate.

1. [**2.5 points**] Prove that if n is an odd integer, then $n^2 + 2n + 3$ is even.

2. [**2.5 points**] Let d and n be integers. Prove that if $d \mid n$ and $d + 1 \mid n$, then $d(d + 1) \mid n$.

3. **[2 parts, 2.5 points each]** *Proof critiques.* Give a critique of each claimed proof below. A *proof critique* addresses the following questions: (1) Is the proof correct? (2) If correct, can the proof be improved in some way? (3) If incorrect, what is/are the error(s)? Can they be fixed, and if so, how?

   (a) **Theorem 1.** *If n is a positive integer and 2^n is odd, then $2n$ is odd.*

   Proof: Let n be a positive integer. Note that 2^{n-1} is an integer since $n \geq 1$. Since $2^n = 2 \cdot 2^{n-1}$, it follows that 2^n is even. Since there are no integers which meet the conditions of the hypotheses, the desired conditional statement is true. $\square$

   (b) **Theorem 2.** *Let a and b be integers. If $a \mid b$, then $|a| \leq |b|$.*

   Proof: Since $a \mid b$, we have that $b = ka$ for some integer k. Taking the absolute value of both sides gives $|b| = |ka| = |k| \cdot |a|$. Note that the product of two positive integers is at least as large as the factors. Therefore $|b| = |k| \cdot |a| \geq |a|$. $\square$