

Name: _____

Directions: Solve the following problems. Give supporting work/justification where appropriate.

1. [6 parts, 1 point each] Define the following statements and open sentences.

$$\begin{array}{ll} P: \text{For each } z \in \mathbb{R}, \text{ we have } z^2 \geq 0. & Q(x): x \in \mathbb{Z}. \\ R(x): x \text{ is an even integer.} & S(A): A \text{ is a finite set.} \end{array}$$

Decide whether the following are true or false; indicate your answer by writing the entire word “true” or the entire word “false”. Give brief justifications for partial credit.

(a) $\sim P$

(b) $S(\mathcal{P}(\mathbb{R}) \cap \mathbb{R})$

(c) $R(3) \Rightarrow S(\mathbb{Z})$

(d) $\sim Q(0) \wedge P \wedge R(6)$

(e) For all x , we have $R(x) \Leftrightarrow Q(\frac{x}{2})$.

(f) $(\sim (S(\emptyset) \Rightarrow R(1))) \vee (P \wedge S(\mathbb{R}^2))$

2. [2 parts, 1 point each] Truth tables and logical equivalence.

(a) Write a truth table for $(P \vee Q) \Rightarrow (P \wedge Q)$.

(b) Give a simple statement which is logically equivalent to $(P \vee Q) \Rightarrow (P \wedge Q)$.

3. [2 parts, 1 point each] Let P , Q , and R be statements. Use the logical operands to express the following statements.

(a) P and Q have the same truth value, but R has the opposite truth value.

(b) If at least two of the statements in $\{P, Q, R\}$ are true, then so is the third.