Name: Solutions

Directions: Solve the following problems. Give supporting work/justification where appropriate.

1. **[6 parts, 1 point each]** Define the following statements and open sentences.

P: For each $z \in \mathbb{R}$, we have $z^2 \geq 0$. $Q(x)$: $x \in \mathbb{Z}$.

$R(x)$: x is an even integer. $S(A)$: A is a finite set.

Decide whether the following are true or false; indicate your answer by writing the entire word "true" or the entire word "false". Brief justification for partial credit.

(a) $\sim P$

TRUE. The square of a real number is non-negative, so P is true and $\sim P$ is FALSE.

(b) $S(\mathcal{P}(\mathbb{R}) \cap \mathbb{R})$

TRUE. Elements of $\mathcal{P}(\mathbb{R})$ are sets. Elements of $\mathbb{R}$ are real numbers. So $\mathcal{P}(\mathbb{R}) \cap \mathbb{R} = \emptyset$.

(c) $R(3) \Rightarrow S(\mathbb{Z})$

Since $R(3)$ is false, the implication is TRUE.

(d) $\{\sim Q(0) \wedge P\} \wedge R(6)$

$\{$FALSE $\wedge$ TRUE$\}$ $\wedge$ TRUE $\rightarrow$ TRUE $\wedge$ TRUE $\rightarrow$ TRUE FALSE

(e) For all x, we have $R(x) \Leftrightarrow Q(\frac{x}{2})$.

TRUE. If x is an even integer, then $\frac{x}{2} \in \mathbb{Z}$. Also, if $\frac{x}{2} \in \mathbb{Z}$, then x is an even integer.

(f) $(\sim (S(\emptyset) \Rightarrow R(1))) \vee (P \wedge S(\mathbb{R}^2))$

$\rightarrow (\sim(\text{TRUE} \Rightarrow \text{FALSE})) \vee (\text{TRUE} \wedge \text{FALSE})$

$\rightarrow (\sim \text{FALSE}) \vee (\text{FALSE}) \rightarrow \text{TRUE} \vee \text{FALSE} \rightarrow$ TRUE

2. **[2 parts, 1 point each]** Truth tables and logical equivalence.

   (a) Write a truth table for $(P \vee Q) \Rightarrow (P \wedge Q)$.

P	Q	$P \vee Q$	$P \wedge Q$	$(P \vee Q) \Rightarrow (P \wedge Q)$
T	T	T	T	T
T	F	T	F	F
F	T	T	F	F
F	F	F	F	T

   (b) Give a simple statement which is logically equivalent to $(P \vee Q) \Rightarrow (P \wedge Q)$.

From the truth table, $[(P \vee Q) \Rightarrow (P \wedge Q)] \equiv \boxed{P \Leftrightarrow Q}$.

3. **[2 parts, 1 point each]** Let P, Q, and R be statements. Use the logical operands to express the following statements.

   (a) P and Q have the same truth value, but R has the opposite truth value.

$$(P \Leftrightarrow Q) \wedge (\sim (Q \Leftrightarrow R))$$

* Many other solns possible *.

   (b) If at least two of the statements in $\{P, Q, R\}$ are true, then so is the third.

Soln 1: $[(P \wedge Q) \vee (P \wedge R) \vee (Q \wedge R)] \Rightarrow [P \wedge Q \wedge R]$

Soln 2: $\sim [(P \wedge Q \wedge \sim R) \vee (P \wedge \sim Q \wedge R) \vee (\sim P \wedge Q \wedge R)]$