Name: ________________________________

Directions: Solve the following problems. Give supporting work/justification where appropriate.

1. **[4 parts, 1 point each]** Let $I = \{\alpha \in \mathbb{R} \colon \alpha \geq 0\}$, and let $A_\alpha = \{(x, y) \in \mathbb{R}^2 \colon y \geq \alpha x\}$ for each $\alpha \in I$. Sketch the following sets, using dashed lines to denote boundaries that are omitted.

   (a) I_0

   (b) I_2

   (c) $\bigcap_{\alpha \in I} A_\alpha$

   (d) $\bigcup_{\alpha \in I} A_\alpha$

2. **[1 point]** Let A be the set of all real numbers that are at distance at most $\frac{1}{3}$ from some integer. For example, A contains 7, $\frac{1}{4}$, and $\frac{2}{3}$, but A does not contain $\frac{1}{2}$ or $\frac{7}{5}$. Express A as an indexed union.

3. **[5 parts, 1 point each]** Determine whether or not the following are statements. In the case of a statement, say whether the statement is true or false.

   (a) It is easier to differentiate a function than it is to integrate it.

   (b) If A and B are nonempty finite sets, then $|A \cup B| > |A|$.

   (c) For all real numbers x, we have $-1 \leq \sin(x) \leq 1$.

   (d) $\mathcal{P}(\mathbb{N}) \cup (\mathbb{R} \times \mathbb{Q})$.

   (e) Every set has at least two distinct subsets.