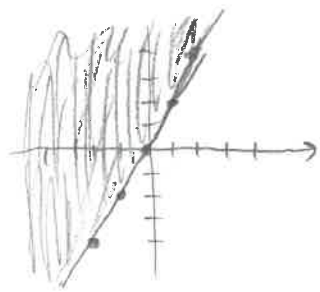
Name: Solutions

Directions: Solve the following problems. Give supporting work/justification where appropriate.

1. **[4 parts, 1 point each]** Let $I = \{\alpha \in \mathbb{R} \colon \alpha \geq 0\}$, and let $A_\alpha = \{(x, y) \in \mathbb{R}^2 \colon y \geq \alpha x\}$ for each $\alpha \in I$. Sketch the following sets, using dashed lines to denote boundaries that are omitted.

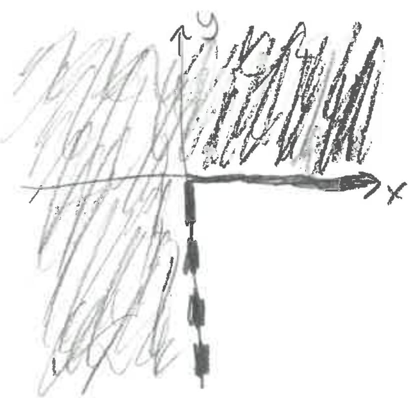
   (a) $A_0 = \{(x,y) \in \mathbb{R}^2 : y \geq 0\}$

   (b) $A_2 = \{(x,y) \in \mathbb{R}^2 : y \geq 2x\}$



   (c) $\bigcap_{\alpha \in I} A_\alpha$

   (d) $\bigcup_{\alpha \in I} A_\alpha$



2. **[1 point]** Let A be the set of all real numbers that are at distance at most $\frac{1}{3}$ from some integer. For example, A contains 7, $\frac{1}{4}$, and $\frac{2}{3}$, but A does not contain $\frac{1}{2}$ or $\frac{7}{5}$. Express A as an indexed union.

$$A = \bigcup_{\alpha \in \mathbb{Z}} \left[\alpha - \tfrac{1}{3}, \alpha + \tfrac{1}{3}\right]$$

3. **[5 parts, 1 point each]** Determine whether or not the following are statements. In the case of a statement, say whether the statement is true or false.

(a) It is easier to differentiate a function than it is to integrate it.

$\boxed{\text{Not a statement}}$ (matter of opinion)

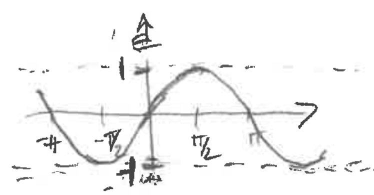
(b) If A and B are nonempty finite sets, then $|A \cup B| > |A|$.

$\boxed{\text{FALSE Statement}}$ If $B \subseteq A$ ~~then~~ (say, $B = \emptyset$), then $|A \cup B| = |A|$.

* Also accepted: $\boxed{\text{Not a statement}}$ since the truth value depends on A and B.

(c) For all real numbers x, we have $-1 \leq \sin(x) \leq 1$.

$\boxed{\text{True Statement}}$



(d) $\mathcal{P}(\mathbb{N}) \cup (\mathbb{R} \times \mathbb{Q})$.

$\boxed{\text{Not A statement}}$ (It is a set)

(e) Every set has at least two distinct subsets.

$\boxed{\text{FALSE Statement}}$ The empty set $\emptyset$ has only one subset (itself)