

Name: Solutions

Directions: Solve the following problems. Give supporting work/justification where appropriate.

1. [6 parts, 1 point each] Let the universe U be $\{1, \dots, 5\}$. Let $A = \{1, 2, 3\}$, $B = \{1, 3\}$, and $C = \{2, 3\}$. Find the following sets.

(a) $B \cap C$

$$\boxed{\{3\}}$$

(b) $C - B$

$$\boxed{\{2\}}$$

(c) $B \cup \bar{A}$

$$\boxed{\{1, 3, 4, 5\}}$$

(d) $(B \times C) - (C \times B)$

$$B \times C = \{(1, 2), (1, 3), (3, 2), (3, 3)\}$$

$$C \times B = \{(2, 1), (2, 3), (3, 1), (3, 3)\}$$

$$(B \times C) - (C \times B) = \boxed{\{(1, 2), (1, 3), (3, 2)\}}$$

(e) $\mathcal{P}(A) - \mathcal{P}(B \cup C)$

$$\mathcal{P}(A) - \mathcal{P}(B \cup C) = \mathcal{P}(A) - \mathcal{P}(A) = \boxed{\emptyset}$$

(f) $\mathcal{P}(A) - (\mathcal{P}(B) \cup \mathcal{P}(C))$

$$\mathcal{P}(A): \text{all subsets of } A$$

$$\mathcal{P}(B) \cup \mathcal{P}(C) = \{X : X \subseteq B \text{ or } X \subseteq C\}$$

so we want $\{X : X \subseteq A \text{ and } X \not\subseteq B \text{ and } X \not\subseteq C\}$,

$$\text{or } \boxed{\{\{1, 2\}, \{1, 2, 3\}\}}.$$

2. [1 point] Are there sets A and B such that $A \cap (A \times B)$ is nonempty? If yes, then give an example. If not, then explain why.

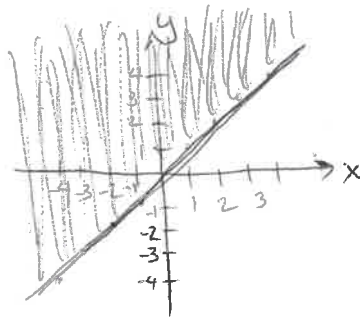
Yes it is possible: let $A = \{0, (0, 1)\}$ and $B = \{1\}$.

Note that $(0, 1) \in A$ and $(0, 1) \in A \times B$.

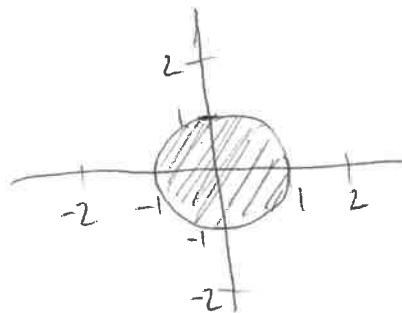
(Many other examples can be given.)

3. [3 parts, 1 point each] Let $A = \{(x, y) \in \mathbb{R}^2 : y \geq x\}$ and $B = \{(x, y) \in \mathbb{R}^2 : x^2 + y^2 \leq 1\}$. Sketch the following sets in the plane, using solid lines to represent boundaries that are in the set, and dashed lines/open circles to represent boundaries that are not in the set.

(a) A



(b) B



(c) $B - A$

