Name: Solutions

Directions: Solve the following problems. Give supporting work/justification where appropriate.

1. **[2 parts, 1 point each]** Let $A = \{1, 2, \varnothing\}$ and $B = \{5, \varnothing\}$. Find the following sets.

   (a) $A \times B$

   $\{(1,5), (1,\varnothing), (2,5), (2,\varnothing), (\varnothing,5), (\varnothing,\varnothing)\}$

   (b) $B^3 = B \times B \times B$

   $= \{(5,5,5), (\varnothing,5,5), (5,\varnothing,5), (\varnothing,\varnothing,5),$
   $(5,5,\varnothing), (\varnothing,5,\varnothing), (5,\varnothing,\varnothing), (\varnothing,\varnothing,\varnothing)\}$

2. **[1 point]** Suppose that $(1, 2) \in A \times B$ and $(2, 3) \in A \times B$. Find two more elements in $A \times B$.

   We know $(1,2) \in A \times B$, and so $1 \in A$ and $2 \in B$.
   Also, $(2,3) \in A \times B$ and so $2 \in A$ and $3 \in B$.
   Hence $\{1,2\} \subseteq A$ and $\{2,3\} \subseteq B$.
   It follows that $\boxed{(1,3)}$ and $\boxed{(2,2)}$ are also elements in $A \times B$.

3. **[3 parts, 1 point each]** Decide whether the following statements are true or false. Write the entire word true or the entire word false to indicate your answer. No explanations or justification required.

   (a) $\{1, 2, 3\} \in \{1, 2, 3\}$ FALSE.

   (b) $\{\mathbb{Q}\} \in \{\mathbb{Z}, \mathbb{Q}, \mathbb{R}\}$

   FALSE.

   (c) $\{\mathbb{Q}\} \subseteq \{\mathbb{Z}, \mathbb{Q}, \mathbb{R}\}$

   TRUE.

4. **[3 parts, 1 point each]** Find the following power sets.

(a) $\mathcal{P}(\{a,b,c\})$

$$\{\emptyset, \{a\}, \{b\}, \{c\}, \{a,b\}, \{a,c\}, \{b,c\}, \{a,b,c\}\}$$

(b) $\mathcal{P}(\{\{1,2\},\{3\}\})$

$$\{\emptyset, \{\{1,2\}\}, \{\{3\}\}, \{\{1,2\},\{3\}\}\}$$

(c) $\mathcal{P}(\{6,7\}\times\{8\})$

$$= \mathcal{P}(\{(6,8), (7,8)\})$$

$$= \{\emptyset, \{(6,8)\}, \{(7,8)\}, \{(6,8),(7,8)\}\}$$

5. **[1 point]** Let $A = \{1,2,3,4\}$. Express the set $\{X \subseteq A\colon |X| \text{ is odd}\}$ by listing its elements between braces.

The subsets of A have sizes in $\{0,1,2,3,4\}$. So we seek subsets of A of size 1 or 3. The set $\{X \subseteq A : |X| \text{ is odd}\}$ is therefore

$$\{\{1\}, \{2\}, \{3\}, \{4\}, \{1,2,3\}, \{1,2,4\}, \{1,3,4\}, \{2,3,4\}\}$$