Name:
Directions: Solve the following problems. Give supporting work/justification where appropriate.

1. [3 points] Suppose $a, b \in \mathbb{Z}$. Prove that if $a b$ is odd, then $a^{2}+b^{2}$ is even.
2. [3 points] Prove that there exist unique real numbers $a$ and $b$ such that the linear function $f$ given by $f(x)=a x+b$ satisfies $f(f(x))=x+1$.
3. [4 points] Let $A, B$, and $C$ be sets. Show that $A \times B \subseteq A \times C$ if and only if $A=\varnothing$ or $B \subseteq C$.
