

Name: Solutions

Directions: Solve the following problems. Give supporting work/justification where appropriate.

1. [2 parts, 1 point each] Express the following sets using a list between braces, using the ellipses if necessary.

(a) $\{\frac{n}{2} : n \in \mathbb{N}\}$

$$\left\{ \frac{1}{2}, 1, \frac{3}{2}, 2, \frac{5}{2}, 3, \frac{7}{2}, \dots \right\} \quad \text{Also OK: } \{0.5, 1, 1.5, 2, 2.5, \dots\}$$

(b) $\{(x, y) : x, y \in \mathbb{Z} \text{ and } x + y = 0\}$

$$\{ \dots, (-2, 2), (-1, 1), (0, 0), (1, -1), (2, -2), \dots \}$$

2. [4 parts, 1 point each] Determine whether the following sets are infinite or finite. If the set is finite, then determine its cardinality.

(a) $\{\{1, 2, (3, 4)\}\}$

Finite, size 1. Only element is the set $\{1, 2, (3, 4)\}$.

(b) $\{\mathbb{Q}\}$

Finite, size 1. Only element is the set \mathbb{Q} .

(c) $\{x \in \mathbb{R} : 0 < x < 1\}$

Infinite. There are many real numbers between 0 and 1.

(d) $\{\emptyset, \{\}, (0, 1), (1, 0)\}$

Finite, size 3. Although $\emptyset = \{\}$, the other objects are distinct.

3. [2 parts, 1 point each] Use set-builder notation to express the following sets in a compact way.

(a) $\{1, 2, 4, 8, 16, 32, 64, \dots\}$

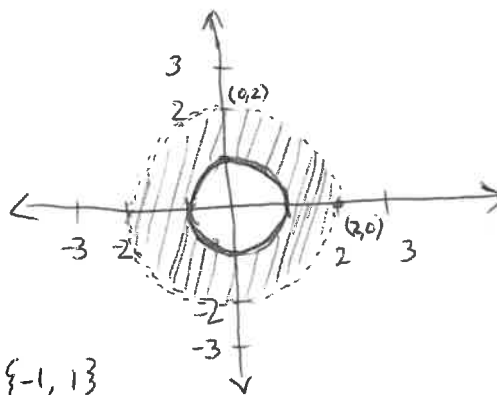
$$\{2^n : n \in \mathbb{Z} \text{ and } n \geq 0\}$$

- (b) The set of all points (x, y) in the interior of the square with vertices $(0, 0)$, $(0, 1)$, $(1, 0)$, and $(1, 1)$.

$$\{(x, y) \in \mathbb{R}^2 : 0 < x, y < 1\}$$

4. [2 parts, 1 point each] Sketch the following sets of points in the x, y -plane \mathbb{R}^2 . Use dashes to denote boundaries that are excluded from the set.

(a) $\{(x, y) \in \mathbb{R}^2 : 1 \leq x^2 + y^2 < 4\}$



(b) $\{(x, y) \in \mathbb{R}^2 : x + y \in \{-1, 1\}\}$

So the condition is

$$x + y = 1 \text{ or } x + y = -1$$

$$y = -x + 1 \quad y = -x - 1$$

These are both lines
with slope -1 .

