

**Directions:** You may work to solve these problems in groups, but all written work must be your own. Unless the problem indicates otherwise, all problems require some justification; a correct answer without supporting reasoning is not sufficient. Submissions must be stapled. See “Guidelines and advice” on the course webpage for more information.

1. Suppose that  $A$  is a set and  $|A| = 84$ . How many subsets of  $A$  have 0 elements? How many have 10 elements? How many have 74 elements?
2. Suppose that  $A$  is a set and there are 330 subsets of  $A$  of size 7. What is  $|A|$ ?
3. Use the binomial theorem to find the coefficient of  $x^4y^8$  in  $(3x - 2y)^{12}$ .
4. An identity.

(a) Using the binomial theorem, show that  $\sum_{k=0}^n \binom{n}{k} = 2^n$ .

(b) By counting the subsets of an  $n$ -element set in two different ways, give a combinatorial proof that  $\sum_{k=0}^n \binom{n}{k} = 2^n$ .

5. Note: this problem is moved to HW9. ~~Prove the following using the method of proof by contradiction.~~
  - ~~(a) Show that  $2^{\frac{1}{3}}$  is irrational.~~
  - ~~(b) Suppose that  $a, b, c \in \mathbb{Z}$ . Show that if  $a^2 + b^2 = c^2$ , then  $a$  or  $b$  is even.~~
  - ~~(c) Prove that there are no integers  $a$  and  $b$  such that  $21a + 30b = 1$ .~~
6. Three people that mutually hate each other are confined to a unit square. (The people are small compared to the square, so they can be modeled as points.) Their buffer is the distance between the closest pair. What is the maximum possible buffer? As usual, be sure to show your work.