Directions: You may work to solve these problems in groups, but all written work must be your own. Unless the problem indicates otherwise, all problems require some justification; a correct answer without supporting reasoning is not sufficient. Submissions must be stapled. See "Guidelines and advice" on the course webpage for more information.

1. For each $n \in \mathbb{N}$, let $A_n = \{2n, 2n+1, \ldots, 3n\}$.
   (a) Find $\bigcup_{z=15}^{20} A_z$ and $\bigcap_{s=15}^{20} A_s$.
   (b) Suppose that $a, b \in \mathbb{N}$ and $a \leq b$. Find $|\bigcup_{k=a}^{b} A_k|$ in terms of a and b.

2. [BP 1.8.{6,8,10}] Recall that for real numbers a and b, we have $[a,b] = \{x \in \mathbb{R} \colon a \leq x \leq b\}$. Find the following sets.
   (a) $\bigcup_{i\in\mathbb{N}}[0, i+1]$
   (b) $\bigcap_{i\in\mathbb{N}}[0, i+1]$
   (c) $\bigcup_{\alpha\in\mathbb{R}}(\{\alpha\} \times [0,1])$
   (d) $\bigcap_{\alpha\in\mathbb{R}}(\{\alpha\} \times [0,1])$
   (e) $\bigcup_{x\in[0,1]}([x,1] \times [0,x^2])$
   (f) $\bigcap_{x\in[0,1]}([x,1] \times [0,x^2])$

3. For each $k \in \mathbb{N}$, let $A_k = \{kn \colon n \in \mathbb{Z}\}$. Find the following sets.
   (a) The examples A_1, A_2, and A_3.
   (b) $\bigcup_{k=1}^{3} A_k$
   (c) $\bigcup_{k=2}^{4} A_k$
   (d) $\bigcap_{k=1}^{3} A_k$
   (e) $\bigcap_{k=1}^{\infty} A_k$
   (f) $\bigcup_{k\in I} A_k$ where $I = \{3, 5, 7, 9, 11, \ldots\}$.

4. [BP 2.1,evens] Decide whether or not the following are statements. In the case of a statement, say if it is true or false, if possible.
   (a) Every even integer is a real number.
   (b) Sets $\mathbb{Z}$ and $\mathbb{N}$.
   (c) Some sets are finite.
   (d) $\mathbb{N} \notin \mathcal{P}(\mathbb{N})$.
   (e) $(\mathbb{R} \times \mathbb{N}) \cap (\mathbb{N} \times \mathbb{R}) = \mathbb{N} \times \mathbb{N}$.
   (f) If the integer x is a multiple of 7, then it is divisible by 7.
   (g) Call me Ishmael.
   (h) If x is an integer, then $x + y$ is also an integer.

5. An infinite series of nested circles and squares are drawn, all sharing a common center point. The outermost circle has radius 1. The space between each circle and the square it circumscribes is shaded. What is the total area of the shaded regions?