

**Directions:** You may work to solve these problems in groups, but all written work must be your own. Unless the problem indicates otherwise, all problems require some justification; a correct answer without supporting reasoning is not sufficient. Submissions must be stapled. See “Guidelines and advice” on the course webpage for more information.

1. Let  $A = \{1, 2, 3\}$  and  $B = \{\sin, \cos\}$ . List the elements of the following sets.

$$\begin{array}{l|l|l} \text{(a) } B \times A & \text{(c) } B \times A \times \emptyset & \text{(e) } \mathcal{P}(B) \\ \text{(b) } B \times (A \times B) & \text{(d) } A \times \{\emptyset\} & \text{(f) } \mathcal{P}(B \times \{a\}) \end{array}$$

2. List the subsets of the following sets.

$$\text{(a) } \{\mathbb{R}, \mathbb{N}, \mathbb{Q}\} \quad | \quad \text{(b) } \emptyset \quad | \quad \text{(c) } \{\{\mathbb{N}\}\}$$

3. Express the set  $\{X \subseteq \mathbb{N} : |X| \leq 1\}$  by listing its elements between braces, using ellipses if necessary.

4. Decide whether the following statements are true or false. Give explanations.

$$\begin{array}{l} \text{(a) } \mathbb{R}^2 \subseteq \mathbb{R}^3 \\ \text{(b) } \{(x, y) \in \mathbb{R}^2 : x^2 - x = 0\} \subseteq \{(x, y) \in \mathbb{R}^2 : x - 1 = 0\} \end{array}$$

5. Suppose that  $|A| = m$  and  $|B| = n$ . Find the given cardinalities.

$$\begin{array}{l|l} \text{(a) } |\mathcal{P}(\mathcal{P}(A))| & \text{(c) } |\mathcal{P}(A) \times \mathcal{P}(B)| \\ \text{(b) } |\mathcal{P}(A \times \mathcal{P}(B))| & \text{(d) } |\{X \subseteq \mathcal{P}(A) : |X| \leq 1\}| \end{array}$$

6. You have two strings of fuse. When lit at one end, each will burn for exactly one hour. The fuses are not necessarily identical, though, and do not burn at a constant rate. All you have with you is a lighter and these two fuses. Can you measure exactly 45 minutes? If so, explain how. If not, explain why.