Directions: You may work to solve these problems in groups, but all written work must be your own. Unless the problem indicates otherwise, all problems require some justification; a correct answer without supporting reasoning is not sufficient. Submissions must be stapled. See "Guidelines and advice" on the course webpage for more information.

1. Exercises from 11.0. In the following, let $A = \{1, 2, 3, 4, 5, 6\}$.
   (a) Write out the relation R that expresses $\mid$ (divides) on A. Then illustrate it with a diagram.
   (b) How many different relations are there on A?
   (c) Let $R = (\mathbb{R} \times \mathbb{R}) - \{(x, x) \colon x \in \mathbb{R}\}$. What familiar relation on $\mathbb{R}$ is this?
2. Exercises from 11.1.
   (a) Let $A = \{a, b, c\}$ and let $R = \{(a, b), (a, c), (c, c), (b, b), (c, b), (b, c)\}$. Is the relation R reflexive? Symmetric? Transitive? If a property does not hold, then explain why.
   (b) Define a relation R on $\mathbb{Z}$ so that $x \; R \; y$ if $|x - y| < 1$. Is R reflexive? Symmetric? Transitive? If a property does not hold, then explain why. What familiar relation is this?
   (c) Prove that the relation $\mid$ (divides) on $\mathbb{Z}$ is reflexive and transitive.
   (d) Give an example of a relation on $\mathbb{Z}$ that is reflexive and symmetric, but not transitive.
3. Critique the following argument.

   Theorem 1. *Let R be a relation on A. If R is symmetric and transitive, then R is reflexive.*

   Proof: We give a direct proof. Suppose that R is symmetric and transitive. We show that R is reflexive. Let $a \in A$ and let b be any other element in A such that $a \; R \; b$. Since R is symmetric, also $b \; R \; a$. Since $a \; R \; b$ and $b \; R \; a$, it follows by the transitivity of R that $a \; R \; a$. Since $a \; R \; a$ for each $a \in A$, it follows that R is reflexive. $\square$
4. Exercises from 11.2.
   (a) Define a relation R on $\mathbb{Z}$ so that $a \; R \; b$ if and only if $4 \mid x + 3y$. Prove that R is an equivalence relation. Describe its equivalence classes.
   (b) Suppose that R and S are equivalence relations on A. Prove that $R \cap S$ is also an equivalence relation on A.
5. Exercises from 12.2.
   (a) Let $D = \mathbb{R} - \{1\}$. Prove that the function $f \colon D \to D$ defined by $f(x) = \left(\frac{x+1}{x-1}\right)^3$ is bijective.
   (b) Consider the function $\theta \colon \{0, 1\} \times \mathbb{N} \to \mathbb{Z}$ defined as $\theta(a, b) = (-1)^a b$. Is θ injective? Is it surjective? Bijective? Explain.
   (c) Consider the function $\theta \colon \mathcal{P}(\mathbb{Z}) \to \mathcal{P}(\mathbb{Z})$ defined as $\theta(X) = \overline{X}$. Is θ injective? Surjective? Bijective? Explain.
6. [Extra Credit] Read section 12.3 and solve the following problem. Suppose that there are five distinguished points inside a unit square. Prove that two of the distinguished points are at distance at most $\frac{\sqrt{2}}{2}$ from each other.