Name: Solutions

Directions: All questions require explanation in English sentences.

1. **[2 parts, 2.5 points each]** Translate the following into formal mathematical language. Then, decide if the statement is true or false. Let E be the set of even integers, let P be the set of primes, and let $D(x, y)$ be "y is an integer multiple of x".

   (a) Whenever the sum of two integers is even, at least one of the summands is even.

   $$\forall x \in \mathbb{Z}.\ \forall y \in \mathbb{Z}.\ \ (x+y \in E) \Rightarrow ((x \in E) \vee (y \in E))$$

   This is false. For example, if $x = 3$ and $y = 5$, then $x+y$ is even but both x and y are odd.

   (b) There is no largest prime.

   $$\neg(\exists p \in P.\ \forall q \in P.\ \ q \leq p)$$

   This is true; since there are infinitely many primes, for each prime p, there is a larger prime q.

2. **[2 parts, 2.5 points each]** Translate the following formal statements into English, in the most natural way possible. Then, decide if the statement is true or false. Let E be the set of even integers, let P be the set of primes, and let $D(x, y)$ be "y is an integer multiple of x".

(a) $\forall x \in \mathbb{Z}. \forall y \in \mathbb{Z}. (x \in E \land x \notin E) \implies (x + y \notin E)$.

The sum of an even and an odd integer is odd.

This is true.

(b) $\exists a, b \in \mathbb{N}. \forall n \in \mathbb{N}. (D(a, n) \land D(b, n)) \implies D(ab, n)$

There exist ~~For some integer~~ positive integers a and b such that

whenever a and b divide any ~~integ~~ positive integer,

so does their product ab.

This is true; for example, with a=1 and b=1. A more interesting example is a=2 and b=3: an even number that is divisible by 3 is also divisible by 6.