Math293E/Math183 **Quiz 7** **Mar. 15, 2016**

Name: Solutions

Directions: All questions require explanation in English sentences.

1. [~~3~~ 4 **points**] Write a definition of the set of integers that are powers of two using set-builder notation.

Many solutions are possible. For example:

$$S = \{ 2^j : j \text{ is a non-negative integer} \}$$

$$S = \{ x \in \mathbb{N} : x = 2^j \text{ for some integer } j \text{ such that } j \geq 0 \}$$

2. [~~2~~ 3 **points**] What is $\mathcal{P}(\mathcal{P}(\mathcal{P}(\varnothing)))$?

$$\mathcal{P}(\varnothing) = \{\varnothing\}$$

$$\mathcal{P}(\mathcal{P}(\varnothing)) = \mathcal{P}(\{\varnothing\}) = \{\varnothing, \{\varnothing\}\}$$

$$\mathcal{P}(\mathcal{P}(\mathcal{P}(\varnothing))) = \mathcal{P}(\{\varnothing, \{\varnothing\}\})$$

$$= \{ \underset{\text{0 elt subsets}}{\varnothing}, \underbrace{\{\varnothing\}, \{\{\varnothing\}\}}_{\text{1 elt subsets}}, \underset{\text{2 elt subsets}}{\{\varnothing, \{\varnothing\}\}} \}$$

3. ~~[3 points] Let $A = \{x \in \mathbb{R}: x^2 - 4 \geq 0\}$ and $B = \{x \in \mathbb{R}: x^2 - 2x - 3 \geq 0\}$. Using set-builder notation, give simple descriptions of $A \cap B$ and $A \cup B$.~~

4. [~~2~~ 3 points] Prove that $\mathcal{P}(A) \cup \mathcal{P}(B) \subseteq \mathcal{P}(A \cup B)$.

Suppose that $X \in \mathcal{P}(A) \cup \mathcal{P}(B)$. Then $X \in \mathcal{P}(A)$ or $X \in \mathcal{P}(B)$, by definition of the union operation.

Since $X \in \mathcal{P}(A)$ or $X \in \mathcal{P}(B)$, it follows that $X \subseteq A$ or $X \subseteq B$ by the definition of power set.

Since $A \subseteq A \cup B$ and $B \subseteq A \cup B$, it follows that $X \subseteq A \cup B$.

Now, by definition of power set, we have that $X \in \mathcal{P}(A \cup B)$.

Since each element of $\mathcal{P}(A) \cup \mathcal{P}(B)$ is also a member of $\mathcal{P}(A \cup B)$, we have $\mathcal{P}(A) \cup \mathcal{P}(B) \subseteq \mathcal{P}(A \cup B)$. $\blacksquare$