Name: Solutions

Directions: All questions require explanation in English sentences.

1. Find simple formulas for the following sums.

   (a) **[2 points]** $1 + 5 + 9 + \cdots + (4n - 3)$

$$\sum_{i=1}^{n} 4i - 3 = 4\sum_{i=1}^{n} i - 3n$$

$$= 4\frac{n(n+1)}{2} - 3n$$

$$= n(2n + 2 - 3)$$

$$= \boxed{n(2n-1)}$$

   (b) **[2 points]** $\sum_{j=n}^{3n} 2j - 1$

Recall: $\sum_{j=1}^{n} (2j-1) = n^2$.

$$= \sum_{j=1}^{3n} (2j-1) - \sum_{j=1}^{n-1} (2j-1)$$

$$= (3n)^2 - (n-1)^2$$

$$= 9n^2 - (n^2 - 2n + 1)$$

$$= \boxed{8n^2 + 2n - 1}$$

(c) [**2 points**] **Caution!** Read carefully: $\sum_{j=1}^{n} 2^n$

Since the summand does not vary with j, we have

$$\sum_{j=1}^{n} 2^n = \underbrace{2^n + 2^n + \ldots + 2^n}_{n \text{ terms}} = \boxed{n2^n}.$$

2. [**2 points**] Prove the following or give a counter example: if A, B, and C are sets, $A \in B$, and $B \in C$, then $A \in C$.

False. For example, if $A = \emptyset$, $B = \{\emptyset\}$, and $C = \{\{\emptyset\}\}$ then $A \in B$ and $B \in C$ but $A \notin C$.

3. [**2 points**] Complete the following sentence to give the definition of the term *subset*. A set A is a *subset* of a set B, denoted $A \subseteq B$ if ....

every element in A is also an element of B