Name: _Solutions_

Directions: All questions require explanation in English sentences.

1. **[4 points]** Consider the statement "I bike to work only if it is sunny." For each of the following situations, decide whether the given statement is true or false.

   (a) It is sunny and I bike to work.

   True

   (b) It is sunny and I do not bike to work.

   True. I promised only that "biking to work" happens only when it is sunny. It could be that I never bike to work.

   (c) It is not sunny and I bike to work.

   FALSE: This contradicts the promise that I bike only exclusively when it is sunny.

   (d) It is not sunny and I do not bike to work.

   True: When I do not bike to work, nothing can be inferred about the weather.

2. **[1 point]** Express the statement "I bike to work only if it is sunny" as a traditional implication of the form "If A, then B".

   If I bike to work, then it is sunny.

3. **[5 points]** Consider the following equation: $y^2 = x^2 + 1$ (*). Decide whether the following statements are true or false. Justify your answers.

(a) For each real number x, there exists a real number y such that the pair x, y satisfies (*).

This is [true]. Suppose that $x = r$. Then if we set $y = \sqrt{r^2 + 1}$, the equation (*) holds.

(b) For each real number y, there exists a real number x such that the pair x, y satisfies (*).

This is [false]. Consider the case that $y = 0$. There is no real number x such that $0 = x^2 + 1$, since $x^2 \geq 0$.

(c) There exists a real number x such that for each real number y, the pair x, y satisfies (*).

This is [false]. Suppose $x = r$. Then the RHS of (*) is the real number $r^2 + 1$, and the LHS varies with y, so not all real numbers y satisfy $y^2 = r^2 + 1$. No single value of x works with all choices of y.

(d) There exists a real number y such that for each real number x, the pair x, y satisfies (*).

This is [false]. Suppose $y = r$. Now the LHS of (*) is a constant and the RHS varies with x. Therefore not all real numbers x satisfy $r^2 = x^2 + 1$. No single value of y works with all choices of x.

(e) If x, y is a pair of real numbers satisfying (*), then $y > x$.

This is [false]. For example, if $x = 0$ and $y = -1$, then x, y satisfies (*) but it is not the case that $y > x$.