

Name: Solutions

Directions: All questions require explanation in English sentences.

1. [5 points] Let $x = \sqrt{6 + \sqrt{6 + \sqrt{6 + \dots}}}$. Determine the value of x .

We may assume that $x = \sqrt{6 + \sqrt{6 + \dots}}$ defines a value

Note that $x = \sqrt{6 + x}$.

Squaring both sides yields $x^2 = 6 + x$, and so

$$x^2 - x - 6 = 0.$$

Factoring, we obtain $(x - 3)(x + 2) = 0$, and so

$$x = -2 \text{ or } x = 3.$$

Since $x = -2$ is inconsistent with $x = \sqrt{6 + x}$,

we conclude that $x = 3$.

2. [5 points] Fill in the blanks and complete the following proof. You may use results proved in class without additional proof.

Theorem 1. Let t be an irrational number and let w be a real number. At least one of the following two numbers is irrational: $t + w$, and $t - w$.

Proof: Suppose for a contradiction that $t + w$ is rational
[circle one: and/or] $t - w$ is rational. Since the rational
numbers are closed under addition, we have that $(t + w) + (t - w)$ is
rational.

[Complete the proof here]

Therefore $2t$ is rational. Since the rationals are closed under multiplication, we have that $(\frac{1}{2})(2t)$ is rational, and so t is rational. This contradicts our ~~assum~~ hypothesis that t is irrational. The contradiction implies that at least one of $t+w$ and $t-w$ is irrational.

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