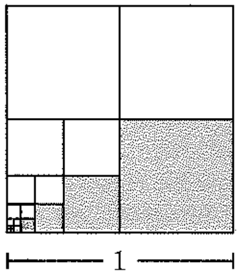
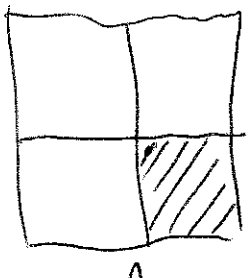
Name: Solutions

Directions: All questions require explanation in English sentences.

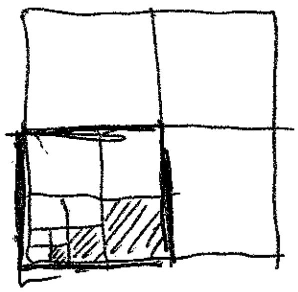
1. **[5 points]** A square is divided into four equal quadrants, and the lower right quadrant is shaded. This operation is iterated forever in the lower left quadrant, as shown below. The outermost square has side length 1. What is the total area of all shaded regions?



Let A be the total shaded area, let A_1 be the area of the lower right quadrant, and let A_2 be the ~~remaining~~ total area of other shaded regions. Note that $A = A_1 + A_2$,



A_1

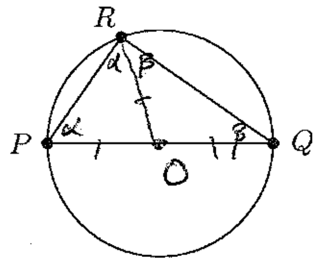


A_2

and $A_1 = \frac{1}{2}\cdot\frac{1}{2} = \frac{1}{4}$. Since regions in A_2 are a scaled down version of the regions in A by a factor of $\frac{1}{4}$, we have $A_2 = \frac{1}{4}A$.

Therefore $A = \frac{1}{4} + \frac{1}{4}A$, or $\frac{3}{4}A = \frac{1}{4}$, so $A = \boxed{\frac{1}{3}}$.

2. **[5 points]** Let PQ be the diameter of a circle C and let R be a point on the circumference of C. Prove that $\angle PRQ = 90°$. (You may use basic facts about triangles without proof.)



Let O be the center of C, and note that OP, OR, and OQ all have the same length. It follows that $\angle OPR = \angle ORP$ and $\angle OQR = \angle ORQ$. Let $\alpha = \angle OPR = \angle ORP$ and $\beta = \angle OQR = \angle ORQ$ as shown above.

Since the degrees of ΔPQR sum to 180, we have

$$\angle RPQ + \angle PQR + \angle QRP = 180$$

$$\alpha + \beta + (\alpha + \beta) = 180$$

$$2(\alpha + \beta) = 180$$

$$\alpha + \beta = 90°$$

Therefore $\angle PRQ = \alpha + \beta = 90°$. ☒