Directions: You may work to solve these problems in groups, but all written work must be your own. **Show your work**; See "Guidelines and advice" on the course webpage for more information.

1. [S 4.2.1] For each of the following sentences, decide whether it is a mathematical statement or not. If it is, decide whether it is True or False. Briefly justify your answers.

   (a) $5 \cdot 25 = 125$.
   (b) We have $\sum_{k\in[n]} k = \frac{n(n+1)}{2}$ for each $n \in \mathbb{N}$.
   (c) For all sets A and B, if $A \subseteq B$, then $B \subseteq A$.
   (d) For all sets A and B, if $A \subseteq B$, then $\mathcal{P}(A) \subseteq \mathcal{P}(B)$.
   (e) Math is cool.
   (f) $1 + 2 = 0$.
   (g) For all $x, y \in \mathbb{Z}$, if xy is even, then x and y are both even.
   (h) For all $x, y \in \mathbb{Z}$, if x and y are both even, then xy is even.
   (i) $1+ = 2$
   (j) $-5 + \mathbb{Z} \geq \pi$
   (k) $x = 7$
   (l) This sentence is true.

2. [S 4.2.5] Come up with a mathematical statement that is True but becomes False when we switch the order of some words.

3. [S 4.3.3] Write an example of a variable proposition $P(x, y)$ such that $\forall x \in \mathbb{Z}.\, \exists y \in \mathbb{R}.\, P(x, y)$ is True but $\forall x \in \mathbb{R}.\, \exists y \in \mathbb{Z}.\, P(x, y)$ is False.

4. [S 4.3.4] For each of the following mathematical statements, write it in symbolic form using quantifiers. (Be sure to define any variable propositions you might need first.) Then, determine whether the statement is True or False.

   (a) There is a real number that is strictly bigger than every integer.
   (b) Every integer has the property that its square is less than or equal to its cube.
   (c) Every natural number's square root is a real number.
   (d) Every subset of $\mathbb{N}$ has the number 3 as an element.

5. [S 4.3.5] Translate each of the following quantified statements into English. Then, determine whether the statement is True or False.

   (a) $\forall x \in \mathbb{N}.\, \exists y \in \mathbb{Z}.\, x + y < 0$
   (b) $\exists x \in \mathbb{N}.\, \forall y \in \mathbb{Z}.\, x + y < 0$
   (c) $\exists A \in \mathcal{P}(\mathbb{Z}).\, \mathbb{N} \subsetneq A \subsetneq \mathbb{Z}$
   (d) Let P be the set of prime numbers. $\forall x \in P.\, \exists t \in \mathbb{Z}.\, x = 2t + 1$
   (e) $\forall a \in \mathbb{N}.\, \exists b \in \mathbb{Z}.\, \forall c \in \mathbb{N}.\, a + b < c$

6. [S 4.4.1] For each of the following statements, write its negation. Which one— the original or the negation— is true?

   (a) $\forall x \in \mathbb{R}.\, \exists n \in \mathbb{N}.\, n > x$
   (b) $\exists n \in \mathbb{N}.\, \forall x \in \mathbb{R}.\, n > x$
   (c) $\forall x \in \mathbb{R}.\, \exists y \in \mathbb{R}.\, y = x^3$
   (d) $\exists y \in \mathbb{R}.\, \forall x \in \mathbb{R}.\, y = x^3$