

Directions: You may work to solve these problems in groups, but all written work must be your own. **Show your work;** See “Guidelines and advice” on the course webpage for more information.

1. Let A and B be finite sets. Either prove or give a counterexample to each of the following statements below.
 - (a) If $A \in B$, then $|A| \leq |B|$.
 - (b) If $A \subseteq B$, then $|A| \leq |B|$.
2. Recall that when n is a nonnegative integer, $[n] = \{1, 2, 3, \dots, n\}$. Write out the elements of $\{a, b\} \times [3]$. How many are there?
3. What is $\emptyset \times \mathbb{Z}$?
4. Let U be a universe of elements, and let A and B be subsets of U . Prove that $\overline{A \cup B} = \overline{A} \cap \overline{B}$.
5. A characterization of a set equation.
 - (a) Give an example of nonempty sets A , B , and C such that $(A \cap B) \cup C = A \cap (B \cup C)$.
 - (b) Give an example of sets A , B , and C such that $(A \cap B) \cup C = A \cap (B \cup C)$ does not hold.
 - (c) Find and prove a characterization for when $(A \cap B) \cup C = A \cap (B \cup C)$ holds.
6. Let $P = \{x \in \mathbb{N} : 1 \leq x \leq 10 \text{ and } x \text{ is prime}\}$. Compute $\sum_{x \in P} (x + 1)$ and $\prod_{x \in P} (x - 1)$.
7. For every $n \in \mathbb{N}$, let $S_n = \{x \in \mathbb{R} : -\frac{1}{n} < x < \frac{1}{n}\}$. Prove that $\bigcap_{n \in \mathbb{N}} S_n = \{0\}$.