Directions: You may work to solve these problems in groups, but all written work must be your own. **Show your work**; See "Guidelines and advice" on the course webpage for more information.

1. [S 1.5.20] Three friends buy a bag of M&M candies. They plan to divide the bag the next day. During the night, one of the friends gets hungry, and decides to eat her share. She divides the candies into 3 equal piles, with 1 M&M left over. She eats her share, and, she impulsively eats the extra. At later points in the night, the two other friends do the same thing. Both times, there is a single extra M&M which is impulsively eaten. The next day, the friends divide the remaining M&M's into three equal parts, and no one mentions the activity the previous night.

   How many M&M's might the bag initially contain? Find all possible answers.

2. [S 1.3.{5,6}] Find all possible values of a such that:
   (a) $x - a$ is a factor of $x^2 + 2ax - 3$.
   (b) $x + a$ is a factor of $x^3 + a$.

3. [S 1.3.7] Let $n \geq 1$. Find a root of $x^n - 1 = 0$ and use it to factor $x^n - 1$ into a polynomial of degree 1 and a polynomial of degree $n - 1$.

4. [S 1.3.10] Some quartic polynomials.
   (a) Find all real solutions to $x^4 - 7x^2 + 18 = 0$.
   (b) Describe how to find all real solutions to $ax^4 + bx^2 + c = 0$.

5. Positive expressions
   (a) Prove that for all real numbers a and b, the inequality $a^2 - 2ab + b^2 \geq 0$ holds.
   (b) Prove that for all real numbers a and b, the inequality $a^2 - ab + b^2 \geq 0$ holds.

6. Let a and b be real numbers such that ab and $a + b$ are integers.
   (a) Find an example that shows that a^2 and b^2 need not be integers.
   (b) Show that $a^2 + b^2$ is an integer.