Directions: You may work to solve these problems in groups, but all written work must be your own. **Show your work**; See "Guidelines and advice" on the course webpage for more information.

1. [S 1.3.8] Determine the value of x defined by $x = \sqrt{2 + \sqrt{2 + \sqrt{2 + \cdots}}}$.
2. [S 1.5.19] You have two strings of fuse. Each will burn for exactly one hour. The fuses are not necessarily identical, though, and do not burn at a constant rate. All you have with you is a lighter and these two fuses. Can you measure exactly 45 minutes? If so, explain how. If not, explain why.
3. Irrational numbers.
   (a) Prove that $1 + \sqrt{2}$ is irrational.
   (b) Prove that $\sqrt{2} + \sqrt{3}$ is irrational.
4. Evaluate the following mathematical argument. Is it clearly written? Does it use correct logic? If not, can it be fixed?

   Theorem 1. *If $p_1, \ldots, p_k$ are the first k primes, then $p_1 p_2 \cdots p_k + 1$ is also a prime.*

   Proof: Let $n = p_1 p_2 \cdots p_k + 1$, and note that $1 = n - p_1 p_2 \cdots p_k$.
   Suppose for a contradiction that some prime p_i less than n divides n. If this were true, then p_i divides both terms on the right hand side of $1 = n - p_1 p_2 \cdots p_k$ and therefore p_i must also divide the left hand side of this equation. Since no prime divides 1, we have a contradiction.
   The contradiction implies that no prime less than n divides n, and therefore n is prime. $\square$