Name: ______________________________

Directions: Show all work. No credit for answers without work. The test has 5 pages, with 10 points per page. **Your lowest scoring page will be dropped.** *Recommendation:* to save on time, decide now which page you will drop and work on the other 4 pages.

1. [**10 points**] A population of animals has a birth rate β and death rate δ that are proportional to P. Recall that this means that $\beta = \lambda_1 P$ and $\delta = \lambda_2 P$ for some constants λ_1 and λ_2. Also, recall that β represents births per unit population per unit time, and δ represents deaths per unit population per unit time.

   (a) Show that $P(t) = \dfrac{P_0}{1 - kP_0 t}$ where $k = \lambda_1 - \lambda_2$.

   (b) When $\lambda_1 > \lambda_2$, the population experiences "doomsday". When does doomsday occur? Find a general formula.

2. [**10 points**] Consider the differential equation $\frac{dP}{dt} = \frac{1}{2}P(11 - P)$ with $P(0) = 3$.

   (a) Using the fact that $P(t)$ is a logistic differential equation, state the solution to $P(t)$. It is not necessary to show any work. *Note:* you may solve this equation directly and copy your answer here, but this will cost you time.

   (b) Use Euler's method with step size $h = 1/3$ to approximate $P(t)$ over the interval $[0, 1]$.

   (c) Other than the error, what is particularly unfortunate about the estimate for $P(2/3)$?

3. [**5 points**] Find the Wronskian of $f_1(x) = 5x$, $f_2(x) = \sin(x)$, and $f_3(x) = e^x$. Are these functions linearly dependent or linearly independent? Explain how you know.

4. [**5 points**] Find general solutions to the following differential equation: $y^{(3)} + 8y'' + 16y' = 0$.

5. [**10 points**] Solve the following initial value problem: $y'' - 6y' + 8y = 0$ with $y(0) = 2$ and $y'(0) = 5$.

6. [**10 points**] A 5 kg-mass is attached to a spring with sprint constant $k = 290$ N/m. Initially, the spring is stretched 3 m from its equilibrium position and is traveling away from equilibrium at 19 m/s. The motion of the spring is damped with damping constant $c = 30$ Ns/m.

   (a) Express the position function of the mass in the form $x(t) = e^{-kt}(A\cos(\omega t) + B\sin(\omega t))$.

   (b) Express the position function of the mass in the form $x(t) = e^{-kt}(C\cos(\omega t - \alpha))$.

   (c) What is the frequency of the motion of the mass? Include units.