Math242 **Quiz 9** **11 April 2012**

Name: Solutions

Directions: Show all work. No credit for answers without work.

1. [**3 points**] Use the translation theorem to find the inverse Laplace transform of $F(s) = \frac{3s+7}{(s+9)^4}$.

$$F(s) = \frac{3s+7}{(s+9)^4} = \frac{3(s+9) - 20}{(s+9)^4}$$

$$\cdot\; G(s) = \frac{3s - 20}{s^4} = 3 \cdot \frac{1}{s^3} - 20 \cdot \frac{1}{s^4}$$

$$g(t) = \frac{3}{2} t^2 + \frac{20}{3!} t^3 = \frac{3}{2} t^2 - \frac{10}{3} t^3$$

$$\cdot\; f(t) = \mathcal{L}^{-1}\{G(s+9)\} = e^{-9t} \mathcal{L}^{-1}\{G(s)\}$$

$$= \boxed{e^{-9t}\left(\frac{3}{2} t^2 - \frac{10}{3} t^3\right)}$$

2. [**3 points**] Find the convolution $t^2 * t$.

$$\int_0^t \tau^2 (t - \tau)\, d\tau$$

$$= \int_0^t t \cdot \tau^2 - \tau^3 \; d\tau$$

$$= \left(\frac{t}{3} \cdot \tau^3 - \frac{\tau^4}{4}\right)_0^t$$

$$= \left(\frac{t^4}{3} - \frac{t^4}{4}\right) - 0 = \boxed{\frac{t^4}{12}}$$

3. [**4 points**] Use the Laplace transform to solve the following IVP.

$$x^{(3)} + x'' - 6x' = 0; x(0) = 0, x'(0) = 1, x''(0) = 1$$

$$\left[s^3X - s^2\cdot 0 - s\cdot 1 - 1\right] + \left[s^2X - s\cdot 0 - 1\right] - 6\left[sX - 0\right] = 0$$

$$s^3X - s - 1 + s^2X - 1 - 6sX = 0$$

$$(s^3 + s^2 - 6s)X - s - 2 = 0$$

$$X = \frac{s+2}{(s^3+s^2-6s)} = \frac{s+2}{s(s^2+s-6)} = \frac{s+2}{s(s+3)(s-2)}$$

$$\frac{s+2}{s(s+3)(s-2)} = \frac{A}{s} + \frac{B}{s+3} + \frac{C}{s-2}$$

$$s+2 = A(s+3)(s-2) + Bs(s-2) + Cs(s+3)$$

- $\underline{s=0}$: $2 = A(-6)$, $A = -\frac{1}{3}$
- $\underline{s=2}$: $4 = C(2)(5)$, $C = \frac{4}{10} = \frac{2}{5}$
- $\underline{s=-3}$: $-1 = B(-3)(-5)$ $B = -\frac{1}{15}$

$$X(s) = -\frac{1}{3}\cdot\frac{1}{s} - \frac{1}{15}\cdot\frac{1}{s+3} + \frac{2}{5}\cdot\frac{1}{s-2}$$

$$x(t) = \boxed{-\frac{1}{3} - \frac{1}{15}e^{-3t} + \frac{2}{5}e^{2t} = \frac{1}{15}\left(-5 - e^{-3t} + 6e^{2t}\right)}$$