Name: Solutions

- **Directions:** Show all work. No credit for answers without work.
- **Solution to part (a) purchased (Max. score = 8):** ________

1. The Laplace Transform.

   (a) [**5 points**] Find the Laplace transform $X(s)$ of the solution $x(t)$ to the following IVP.

$$x'' + 8x' + 15x = 1; x(0) = 2, x'(0) = -3$$

Express $X(s)$ as a ratio $\frac{F(s)}{G(s)}$ ~~for sme functions~~ of polynomials $F(s)$ and $G(s)$.

$$\mathcal{L}\{x''\} + \mathcal{L}\{8x'\} + \mathcal{L}\{15x\} = \mathcal{L}\{1\}$$

$$[s^2X - sx(0) - x'(0)] + 8[\cancel{8}\, sX - x(0)] + 15X = \frac{1}{s}$$

$$[s^2X - 2s + 3] + 8[sX - 2] + 15X = \frac{1}{s}$$

$$(s^2 + 8s + 15)X - 2s + 3 - 16 = \frac{1}{s}$$

$$(s+3)(s+5)X = \frac{1}{s} + 2s + 13$$

$$X = \frac{\frac{1}{s} + 2s + 13}{(s+3)(s+5)} \cdot \frac{s}{s}$$

$$X = \frac{1 + 2s^2 + 13s}{s(s+3)(s+5)}$$

$$\boxed{X = \frac{2s^2 + 13s + 1}{s(s+3)(s+5)}}$$

(b) [**5 points**] Find the inverse Laplace transform of $X(s)$. **Note:** you may purchase $X(s)$. If you purchase $X(s)$, then your quiz will be graded as usual, except that your maximum score will be capped at 8 points out of 10.

$$\frac{2s^2+13s+1}{s(s+3)(s+5)} = \frac{A}{s} + \frac{B}{s+3} + \frac{C}{s+5}$$

$$A(s+3)(s+5) + Bs(s+5) + Cs(s+3) = 2s^2+13s+1$$

- $\underline{s=0:}$ $\quad A \cdot 3 \cdot 5 = 1, \quad A = \frac{1}{15}$

- $\underline{s=-3:}$ $\quad B \cdot (-3)(2) = 2(-3)^2 + 13(-3) + 1 = -20$

$$B = \frac{-20}{-6} = \frac{10}{3}$$

- $\underline{s=-5:}$ $\quad C(-5)(-2) = 2(-5)^2 + 13(-5) + 1 = -14$

$$C = \frac{-14}{10} = \frac{-7}{5}$$

- $X(s) = \frac{1}{15} \cdot \frac{1}{s} + \frac{10}{3} \cdot \frac{1}{s+3} - \frac{7}{5} \cdot \frac{1}{s+5}$

- $\boxed{x(t) = \frac{1}{15} + \frac{10}{3} e^{-3t} - \frac{7}{5} e^{-5t}}$

$$\boxed{= \frac{1}{15}\left(1 + 50e^{-3t} - 21e^{-5t}\right)}$$