Name: Solutions

Directions: Show all work. No credit for answers without work. This quiz has 12 points but will be graded out of 10.

1. **[3 points]** Find the general solution to $y'' + 5y' = e^{4x}$.

$r^2 + 5r = 0$

$r(r+5) = 0$

$r = 0, \; r = -5$

$y_c = c_1 + c_2 e^{-5x}$

RHS: e^{4x}

~~$4e^{4x}$~~

$y_p = Ae^{4x}$

$y_p' = 4Ae^{4x}, \quad y_p'' = 16Ae^{4x}$

$16Ae^{4x} + 5 \cdot 4Ae^{4x} = e^{4x}$

$[e^{4x}]: \quad 16A + 20A = 1$

$36A = 1$

$A = \frac{1}{36}$

$y = y_c + y_p$

$= \boxed{c_1 + c_2 e^{-5x} + \frac{1}{36} e^{4x}}$

2. **[3 points]** Find a particular solution to $y'' - y = e^x + 7$.

$r^2 - 1 = 0$

$(r+1)(r-1) = 0$

$y_c = c_1 e^{-x} + c_2 e^{x}$

RHS: e^x ; 7

~~e^x~~ ; ~~$\emptyset$~~

repeated in y_c

$y_p = Axe^x + B$

$y_p' = A(e^x + xe^x)$

$y_p'' = A(e^x + e^x + xe^x)$

$= 2Ae^x + Axe^x$

$(2Ae^x + \cancel{Axe^x}) - (\cancel{Axe^x} + B) = e^x + 7$

$2Ae^x - B = e^x + 7$

$[e^x]: \quad 2A = 1, \quad A = \frac{1}{2}$

$[1]: \quad -B = 7, \quad B = -7$

$\boxed{y_p = \frac{1}{2}xe^x - 7}$

3. [**2 points**] State the definition of the Laplace transform of $f(t)$.

$$\mathcal{L}\{f(t)\} = \int_0^\infty e^{-st} f(t)\, dt$$

4. [**2 points**] Find the Laplace transform of $7t - 3e^{2t}$.

$$\mathcal{L}\{7t - 3e^{2t}\} = 7\,\mathcal{L}\{t\} - 3\,\mathcal{L}\{e^{2t}\}$$

$$= \boxed{7 \cdot \frac{1}{s^2} - 3 \cdot \frac{1}{s-2}, \quad s > 2}$$

5. [**2 points**] Find the inverse Laplace transform of $\frac{3}{s-6}$.

$$\mathcal{L}^{-1}\left\{\frac{3}{s-6}\right\} = \boxed{3e^{6t}}$$