

Name: \_\_\_\_\_

**Directions:** Show all work. No credit for answers without work.

1. [**3 points**] Solve the following initial value problem:  $\frac{dx}{dt} = 7x(x - 13)$ , and  $x(0) = 17$ .

2. [**2 points**] Recall the logistic differential equation  $\frac{dx}{dt} = kx(M - x)$ . A population of cows satisfying the logistic equation initially has 4000 members and is growing at a rate of 3 cows per day. The environment can support a population of 10,000 cows.

(a) Solve for  $k$  in the logistic differential equation.

(b) Explicitly give the formula for  $x(t)$ .

3. [**3 points**] Find the equilibrium solutions to  $\frac{dx}{dt} = x(x^2 - 4)$ . Use a phase diagram to classify each equilibrium solution as stable, semi-stable, or unstable.

4. [**2 points**] Give the bifurcation diagram for the differential equation  $\frac{dx}{dt} = x + kx^3$ ; this is the plot of all points  $(k, c)$  such that  $x = c$  is an equilibrium solution to  $\frac{dx}{dt} = x + kx^3$ .