Name: _____

Directions: Show all work. No credit for answers without work.

1. [**2 points**] Consider the general first-order differential equation $\frac{dy}{dx} = f(x, y)$ with initial value $y(a) = b$. State the theorem which gives conditions under which a solution exists and is unique.

2. [**2 parts, 1 point each**] For each of the following initial value problems, determine whether the above theorem guarantees existence and uniqueness.

   (a) $\frac{dy}{dx} = \frac{x^3(y-1)}{x+1}$, and $y(2) = 1$.

   (b) $\frac{dy}{dx} = \frac{x^3\sqrt{y-1}}{x+1}$, and $y(2) = 1$.

3. [**3 points**] Solve the following initial value problem: $\dfrac{dy}{dx} = \dfrac{x^2}{e^y}$, and $y(3) = 0$.

4. [**3 points**] Find the general solution to $\dfrac{dy}{dx} = y\sin(x) + \sin(x)$.