Name: Solution

Directions: Show all work. No credit for answers without work.

1. [**5 points**] The general solution to $xy' = x^2 + y$ is one of the following:

$$y = Cx^2 + x \mid y = x^2 + Cx \mid y = C(x^2 + x) \mid y = x^2 + x + C$$

(a) Which of the above four function families is the general solution?

$y = Cx^2 + x$

$y' = 2Cx + 1$

$xy' \stackrel{?}{=} x^2 + y$

$2Cx^2 + x \stackrel{?}{=} x^2 + Cx^2 + x$ No

$y = x^2 + Cx$

$y' = 2x + C$

$xy' \stackrel{?}{=} x^2 + y$

$2x^2 + Cx \stackrel{?}{=} x^2 + x^2 + Cx$ YES

$y = C(x^2 + x)$

$y' = C(2x + 1)$

$xy' \stackrel{?}{=} x^2 + y$

$C(2x^2 + x) \stackrel{?}{=} x^2 + C(x^2 + x)$ No

$y = x^2 + x + C$

$y' = 2x + 1$

$xy' \stackrel{?}{=} x^2 + y$

$2x^2 + x \stackrel{?}{=} x^2 + x^2 + x + C$ No

So, $\boxed{y = x^2 + Cx}$ is the general solution.

(b) Solve the initial value problem: $xy' = x^2 + y$, and $y(2) = 0$.

$0 = (2)^2 + C \cdot 2$

$-4 = 2C$

$C = -2$

Soln: $\boxed{y = x^2 - 2x}$

2. [**5 points**] A ball is thrown upward from the top of a tall building at 15m/s. If the ball hits the ground at 70m/s, how tall is the building? (Recall that the acceleration due to gravity is about 9.8m/s^2.)

- Building height $= x_0$.
- $\frac{dv}{dt} = -9.8$
- $v = -9.8t + C$ $\quad [t=0;\ v=15;\ \text{so } C=15]$
- $v = -9.8t + 15$
- $\frac{dx}{dt} = -9.8t + 15$
- $x = \frac{-9.8}{2}t^2 + 15t + C \quad [t=0,\ x=x_0;\ \text{so } C=x_0]$
- $x = \frac{-9.8}{2}t^2 + 15t + x_0$

- Let t_* be the time of impact. Then:

(a) $-70 = -9.8t_* + 15$

(b) $0 = \frac{-9.8}{2}t_*^2 + 15t_* + x_0$

Solve (a): $t_* = \frac{85}{9.8}$

Solve (b):

Solve (b):

$$0 = \frac{-9.8}{2}\left(\frac{85}{9.8}\right)^2 + 15\left(\frac{85}{9.8}\right) + x_0$$

$$x_0 = \frac{(85)^2}{2 \cdot 9.8} - \frac{15 \cdot 85}{9.8}$$

$$\approx \boxed{238.52 \text{ m}}$$