

Name: Solution

Directions: Show all work. No credit for answers without work.

1. [3 parts, 2 points each] Let A , B , C , and k be constants. In the following, $\arcsin(x)$ is the inverse of $\sin(x)$, so that $\arcsin(\sin(x)) = x$ and similarly, $\arctan(x)$ is the inverse of $\tan(x)$. Differentiate the following with respect to x .

(a) $y = (Ax^2 + Bx + C)^5$

$$y' = 5(Ax^2 + Bx + C)^4 \cdot \frac{d}{dx} [Ax^2 + Bx + C]$$

$$= \boxed{5(Ax^2 + Bx + C) \cdot (2Ax + B)}$$

(b) $y = \tan(Ax) + e^{Bx} + x^{2k}$

$$y' = \boxed{\sec^2(Ax) \cdot A + B e^{Bx} + (2k) x^{2k-1}}$$

(c) $y = \arcsin(kx) + \arctan(kx)$

$$y' = \frac{1}{\sqrt{1-(kx)^2}} \cdot k + \frac{1}{1+(kx)^2} \cdot k$$

$$= \boxed{\frac{k}{\sqrt{1-(kx)^2}} + \frac{k}{1+(kx)^2}}$$

2. [2 parts, 2 points each] Solve the following integrals.

(a) $\int x^2 + x(5x^2 + 12)^8 dx$

$$= \int x^2 dx + \frac{1}{10} \int (5x^2 + 12)^8 \cdot 10x dx$$

$$= \frac{x^3}{3} + \frac{1}{10} \int u^8 du$$

$$= \frac{x^3}{3} + \frac{1}{10} \frac{u^9}{9} + C$$

$$= \boxed{\frac{x^3}{3} + \frac{1}{90} (5x^2 + 12)^9 + C}$$

$$\begin{array}{l} u = 5x^2 + 12 \\ du = 10x dx \end{array}$$

(b) $\int \sin(x) \cos(x) dx$

$$\begin{array}{l} u = \sin(x) \\ du = \cos(x) dx \end{array}$$

$$\int \sin(x) \cos(x) dx$$

$$= \int u du$$

$$= \frac{u^2}{2} + C$$

$$= \boxed{\frac{\sin^2(x)}{2} + C}$$