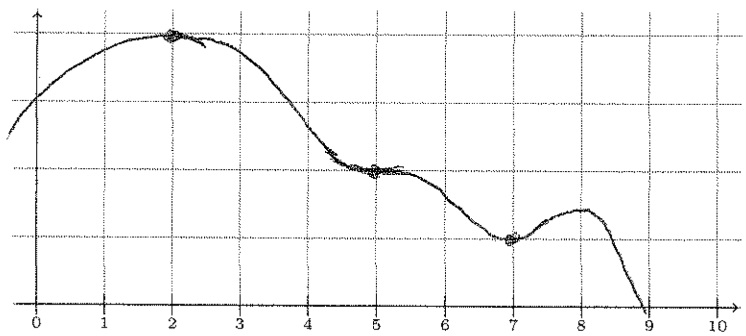
Name: Solutions

Directions: Show all work. No credit for answers without work.

1. [**10 points**] Draw a single graph that has each of the following three properties:

   - a global maximum at $x = 2$,
   - a critical point which is neither a local minimum nor a local maximum at $x = 5$, and
   - a local minimum which is not a global minimum at $x = 7$.

HW8, 4.3 #5,7,9



2. [**10 points**] Find the exact global maximum and global minimum values of $f(x) = xe^{-2x}$ over the closed interval $[-1, 1]$. (Decimal approximations with appropriate work are worth partial credit.)

$$f'(x) = \frac{d}{dx}\left[xe^{-2x}\right]$$

$$= \frac{d}{dx}[x]e^{-2x} + x\frac{d}{dx}\left[e^{-2x}\right]$$

$$= e^{-2x} - 2xe^{-2x}$$

$$= e^{-2x}(1-2x)$$

Critical pts: $e^{-2x}(1-2x) = 0$

$e^{-2x} = 0$ or $1 - 2x = 0$

No soln $\quad x = \frac{1}{2}$

Check:

- $f(-1) = (-1)e^{-2(-1)} = -\frac{1}{e^2}$
- $f(\frac{1}{2}) = \frac{1}{2}e^{-2(\frac{1}{2})} = \frac{1}{2e}$
- $f(1) = 1e^{-2} = \frac{1}{e^2}$

Global min: $-\frac{1}{e^2}$

Global max: $\frac{1}{2e}$

HW8, 4.3 #25

[HW8, 4.3 #7]

3. [**2 parts, 4 points each**] Mike owns a small business that produces desks. His total cost $C(q)$ (in dollars) to produce q desks is given by $C(q) = q^2 + 200q + 400$.

   (a) Find the marginal cost function and the average cost function.

$$MC = 2q + 200$$

$$AC = \frac{C(q)}{q} = \frac{q^2 + 200q + 400}{q} = q + 200 + \frac{400}{q}$$

   (b) Find the production level that minimizes Mike's average cost. What is the minimum possible average cost?

$$AC = MC$$

$$q + \cancel{200} + \frac{400}{q} = 2q + \cancel{200}$$

$$\frac{400}{q} = q$$

$$400 = q^2$$

$$q = \pm 20$$

$$AC(20) = 20 + 200 + \frac{400}{20} = 240$$

• So, AC is minimized when $\boxed{\text{prod. level is 20 desks,}}$ where the average cost is $\boxed{\$240}$

[In Class] [Book: 4.5 Example 1]

4. [**4 points**] Fill in the blanks: on the graph of the cost function $C(q)$, the average cost at production level q is represented by the slope of the line joining $(0, 0)$ and $(q, C(q))$.

[HW8, 4.4 #5]

5. [**2 parts, 4 points each**] A company that produces books has cost function $C(q)$ (in dollars) and revenue function $R(q)$ (in dollars). Currently, the production level is $q = 70$ books, and $C'(70) = 23$ and $R'(70) = 21$.

   (a) Estimate the change in profit that results from producing the 71st book.

$$MP = MR - MC = 21 - 23 = -2$$

Company will lose \$2 by producing the 71st book.

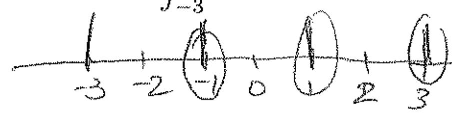
   (b) Should the company increase production, decrease production, or leave production unchanged?

The company should $\boxed{\text{reduce production}}$, since $MC > MR$.

[HW9, 5.2 #1, #17(b)] HW5.1

6. [**6 points**] Give the Right Hand Sum approximation to $\int_{-3}^{3} x(x+1)\,dx$ with $n = 3$.

$\Delta x = \frac{3-(-3)}{3} = \frac{6}{3} = 2$



$RHS = 2(-1)\cancel{(-1+1)} + 2(1)(2) + 2(3)(4)$

$= 4 + 24 = \boxed{28}$

[HW9 5.3 #3]

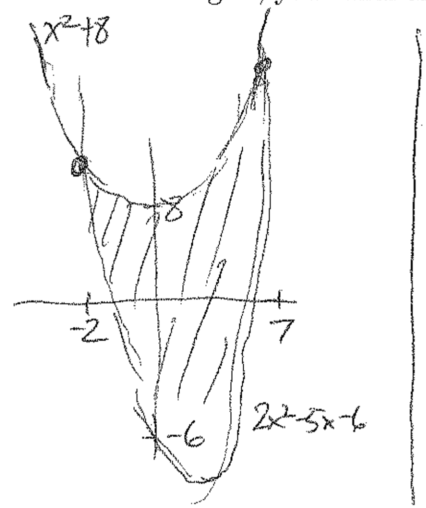
7. [**6 points**] Express the area bounded by the curves $y = 2x^2 - 5x - 6$ and $y = x^2 + 8$ as a definite integral. You do not need to solve this integral; your final answer is the integral.

$2x^2 - 5x - 6 = x^2 + 8$

$x^2 - 5x - 14 = 0$

$(x-7)(x+2) = 0$

$x = -2,\ x = 7$



$\boxed{\text{Area} = \int_{-2}^{7} (x^2+8) - (2x^2-5x-6)\,dx}$

$= \boxed{\int_{-2}^{7} -x^2 + 5x + 14\,dx}$

8. [**2 parts, 4 points each**] A printer is able to produce pages faster as it warms up. After t minutes have elapsed since starting a print job, the printer produces pages at a rate of $4t$ pages per minute.

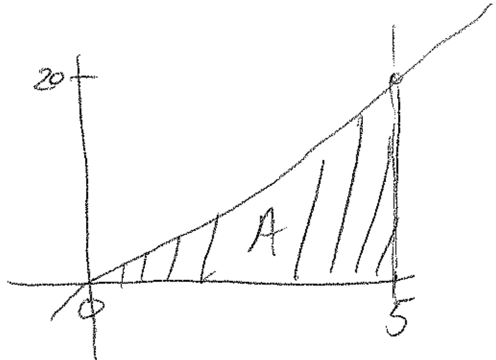
HW9 [5.4 #7]

(a) Express the number of pages printed during the first 5 minutes as a definite integral.

$\int_0^5 4t\,dt$

[HW9 5.2 #18]

(b) Use the graphical interpretation of the definite integral to determine the number of pages printed during the first 5 minutes exactly. (Your answer must demonstrate that you understand the graphical interpretation of the definite integral.)



$\int_0^5 4t\,dt = A$

$= \frac{1}{2}bh$

$= \frac{1}{2}(5)(20) = \boxed{50 \text{ pages}}$

9. [**10 parts, 2 points each**] Evaluate the following.

(a) $\int 2\,dx$

$= \boxed{2x + C}$

(b) $\int 0\,dz$

$= \boxed{C}$

(c) $\int 2t^3 - 6t^2\,dt$

$= \frac{2}{4}t^4 - \frac{6}{3}t^3 + C$

$= \boxed{\frac{1}{2}t^4 - 2t^3 + C}$

[HW #10 7.1]

(d) $\int e^{-2x}\,dx$

$= \boxed{-\frac{1}{2}e^{-2x} + C}$

(e) $\int r^{-1}\,dr \quad = \int \frac{1}{r}dr$

$= \boxed{\ln|r| + C}$

(f) $\int \frac{1}{\sqrt{y}}\,dy \quad = \int y^{-\frac{1}{2}}dy$

$= \frac{y^{\frac{1}{2}}}{\frac{1}{2}} + C$

$= \boxed{2y^{\frac{1}{2}} + C}$

(g) $\int x^{\ln(2)}\,dx$

$= \boxed{\frac{x^{\ln(2)+1}}{\ln(2)+1} + C}$

(h) $\int t(5t^4 + 3)\,dt \quad = \int 5t^5 + 3t\,dt$

$= \boxed{\frac{5}{6}t^6 + \frac{3}{2}t^2 + C}$

(i) $\int \frac{3s^2 + 7}{s}\,ds \quad = \int 3s + \frac{7}{s}ds$

$= \boxed{\frac{3}{2}s^2 + 7\ln|s| + C}$

(j) $\int (e^{3z} + 2)^2\,dz \quad = \int (e^{3z})^2 + 4e^{3z} + 4dz$

$= \int e^{6z} + 4e^{3z} + 4dz$

$= \boxed{\frac{1}{6}e^{6z} + \frac{4}{3}e^{3z} + 4z + C}$

10. [**4 parts, 5 points each**] Evaluate the following.

(a) $\int (6t+5)(3t^2+5t)^{14}\,dt$

$w = 3t^2+5t$

$\frac{dw}{dt} = 6t+5$

$dw = (6t+5)dt$

$\int (6t+5)(3t^2+5t)^{14}dt = \int w^{14}dw$

$= \frac{w^{15}}{15} + C = \boxed{\frac{(3t^2+5t)^{15}}{15} + C}$

[HW #10 7.2]

(b) $\int \frac{(\ln z)^5 + (\ln z)^2}{z}\,dz$

$w = \ln z$

$\frac{dw}{dz} = \frac{1}{z}$

$dw = \frac{1}{z}dz$

$= \int ((\ln z)^5 + (\ln z)^2) \cdot \frac{1}{z}dz$

$= \int w^5 + w^2\, dw$

$= \frac{w^6}{6} + \frac{w^3}{3} + C$

$= \boxed{\frac{(\ln z)^6}{6} + \frac{(\ln z)^3}{3} + C}$

(c) $\int \frac{x}{x^2+1}\,dx$ $= \int \frac{1}{x^2+1} \cdot \frac{1}{2} \cdot 2x\,dx$

$w = x^2+1$

$\frac{dw}{dx} = 2x$

$dw = 2x\,dx$

$= \int \frac{1}{2} \cdot \frac{1}{w}\, dw$

$= \frac{1}{2}\ln|w| + C$

$= \boxed{\frac{1}{2}\ln|x^2+1| + C}$

(d) $\int \frac{e^{\sqrt{y}}}{\sqrt{y}}\,dy$ $= \int 2e^{\sqrt{y}} \cdot \frac{1}{2\sqrt{y}}\,dy$

$w = y^{\frac{1}{2}}$

$\frac{dw}{dy} = \frac{1}{2}y^{-\frac{1}{2}}$

$dw = \frac{1}{2\sqrt{y}}dy$

$= \int 2e^w\, dw$

$= 2e^w + C$

$= \boxed{2e^{\sqrt{y}} + C}$