Name: Solutions

Directions: Show all work. No credit for answers without work.

1. Let $f(x) = x^4 - 3x^3 + 18x + 1$.

   (a) [**5 points**] Find $f''(x)$ in factored form.

   $$f'(x) = 4x^3 - 9x^2 + 18$$
   $$f''(x) = 12x^2 - 18x$$
   $$= 6x(2x-3)$$

   (b) [**5 points**] Make a sign chart for the second derivative $f''(x)$.

   $$6x(2x-3) = 0$$
   $$6x = 0 \text{ or } 2x - 3 = 0$$
   $$x = 0 \text{ or } x = \frac{3}{2}$$

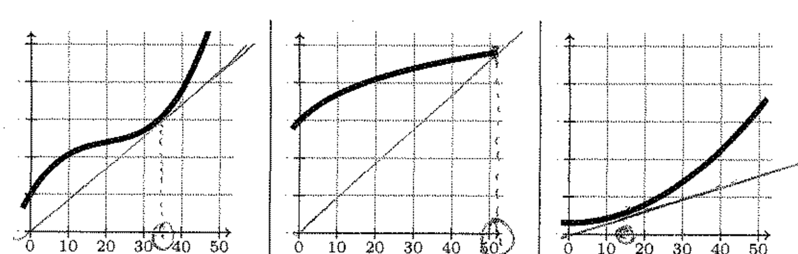
   sign f'': ++ — — ++

   0 3/2

   (c) [**3 points**] Find all inflection points of $f(x)$.

   $$x = 0, \quad x = \frac{3}{2}$$

2. [**3 parts, 2 points each**] The graphs of three cost functions are shown below. For each cost function, determine approximately which production level in the interval $[0, 50]$ minimizes average cost.



(a) (b) (c)

34 50 15

3. [**6 points**] The total cost C (in thousands of dollars) for a company to produce q units of a good is given by the cost function $C(q) = 21q^2 + 80q + 100$. The total revenue R (in thousands of dollars) for a company to produce q units of a good is given by the revenue function $R(q) = 1052q$. Find the production level that maximizes profit.

$$MC = C'(q) = 42q + 80$$

$$MR = R'(q) = 1052$$

Max profit: $MC = MR$

$$42q + 80 = 1052$$

$$42q = 972$$

$$q \approx \boxed{23.14}$$

4. [**3 parts, 6 points each**] A company produces cars.

(a) When the price of a car is 20 thousand dollars, 46 million vehicles are sold. When the price is 24 thousand dollars, 41 million vehicles are sold. Assuming the demand curve is linear, find the demand q in millions of vehicles as a function of the price p of the car in thousands of dollars.

$$m = \frac{\Delta q}{\Delta p} = \frac{41-46}{24-20} = \frac{-5}{4} = -1.25$$

$$q - q_0 = m(p - p_0)$$

$$q - 46 = -1.25(p - 20)$$

$$q = -1.25p + 25 + 46$$

$$\boxed{q = -1.25p + 71}$$

(b) Find the revenue R as a function of price p.

$$R = q \cdot p = (-1.25p + 71)p = -1.25p^2 + 71p$$

(c) Determine the price and production level that maximizes the company's revenue.

$$R'(p) = -2.5p + 71$$

$$-2.5p + 71 = 0$$

$$-2.5p = -71$$

$$p = 28.4$$

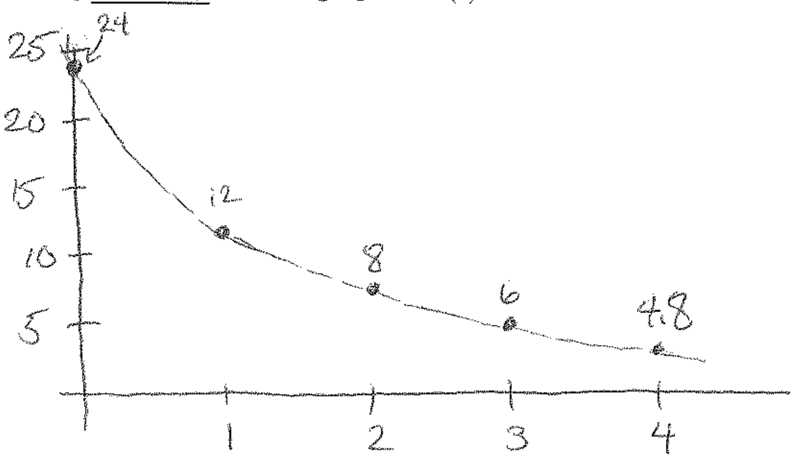
$$q = -1.25(28.4) + 71$$

$$= 35.5$$

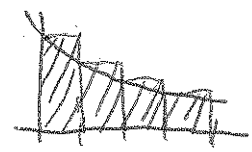
Revenue maximized when price is $\boxed{\$28,400}$ and production is $\boxed{\text{35.5 million vehicles}}$.

5. Your velocity v (in m/sec) at time t (in seconds) is given by the function $v(t) = \frac{24}{1+t}$.

(a) [**7 points**] Carefully draw a graph of $v(t)$ from $t = 0$ to $t = 4$ seconds.



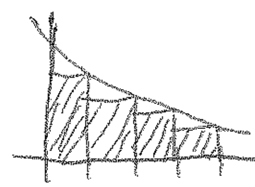
(b) [**5 points**] Using 4 rectangles and the left hand rule, estimate the distance traveled between time $t = 0$ and $t = 4$. Is your estimate smaller or larger than the true distance traveled?



LHS: $1 \cdot 24 + 1 \cdot 12 + 1 \cdot 8 + 1 \cdot 6$

$= \boxed{50 \text{ m}}$. This estimate is $\boxed{\text{larger}}$ than true distance.

(c) [**5 points**] Using 4 rectangles and the right hand rule, estimate the distance traveled between time $t = 0$ and $t = 4$. Is your estimate smaller or larger than the true distance traveled?

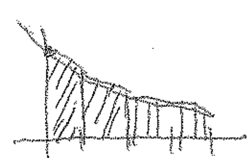


RHS: $1 \cdot 12 + 1 \cdot 8 + 1 \cdot 6 + 1 \cdot 4.8$

$= \boxed{30.8 \text{ m}}$ This estimate is $\boxed{\text{smaller}}$ than true distance.

(d) [**3 points**] Average your estimates in parts (b) and (c) to obtain a more accurate estimate. Is this estimate smaller or larger than the true distance traveled?

Avg $= \frac{1}{2}(50 + 30.8) = \boxed{40.4 \text{ m}}$



This estimate is $\boxed{\text{larger}}$ than the true distance traveled.

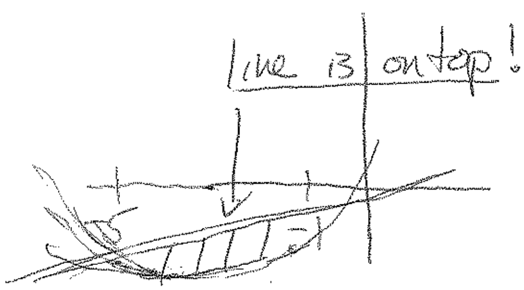
6. [**7 points**] Express the area of the region bounded by $y = x^2 + 8x + 1$ and $y = 2x - 4$ as a definite integral. You do not need to solve the integral; your answer is the integral.

Find intersection pts: $x^2 + 8x + 1 = 2x - 4$

$$x^2 + 6x + 5 = 0$$

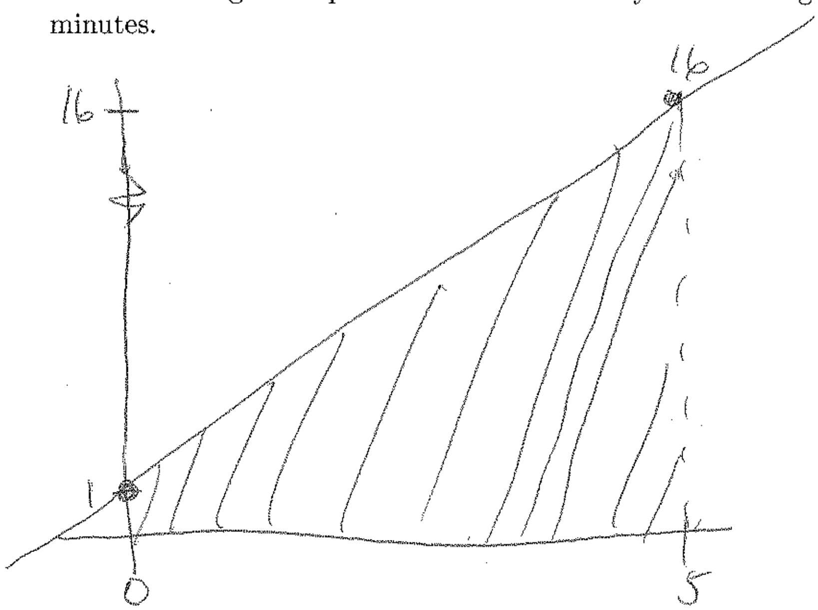
$$(x+1)(x+5) = 0$$

$$x = -1 \text{ or } x = -5$$



$$\text{Area} = \boxed{\int_{-1}^{-5} (2x-4) - (x^2+8x+1)\, dx}$$

7. [**7 points**] A gasoline pump is activated. After t minutes, the pump dispenses gasoline at a rate of $3t + 1$ gallons per minute. Find exactly how much gasoline has been pumped after 5 minutes.



$$\text{GAS} = \int_0^5 3t + 1\, dt$$

$$= \text{area of trap}$$

$$= \tfrac{1}{2}(1+16)\cdot 5$$

$$= \tfrac{1}{2}\cdot 17 \cdot 5 = \boxed{42.5 \text{ gal}}$$

8. [**3 points**] Fill in the blank. Whereas the definite integral $\int_a^b f(x)\, dx$ is a single numerical value, the indefinite integral $\int f(x)\, dx$ is <u>a family of functions</u>.

9. [**10 parts, 2 points each**] Evaluate the following indefinite integrals.

(a) $\int 4\,dx$

$\boxed{4x + C}$

(b) $\int \frac{7}{x^3}\,dx$ $= \int 7x^{-3}\,dx$

$\boxed{-\frac{7}{2}x^{-2} + C}$

(c) $\int e^{-x}\,dx$

$\boxed{-e^{-x} + C}$

(d) $\int 6x^2 - 5x + 1\,dx$

$6\frac{x^3}{3} - 5\frac{x^2}{2} + x$

$= \boxed{2x^3 - \frac{5}{2}x^2 + x + C}$

(e) $\int \frac{2}{x}\,dx$

$= \boxed{2\ln|x| + C}$

(f) $\int \frac{5x+1}{x}\,dx$ $= \int 5 + \frac{1}{x}\,dx$

$= \boxed{5x + \ln|x| + C}$

(g) $\int e^2\,dx$

$= \boxed{e^2 x + C}$

(h) $\int x^{\sqrt{3}}\,dx$

$\boxed{\frac{x^{\sqrt{3}+1}}{\sqrt{3}+1} + C}$

(i) $\int (e^{3x} + 1)^2\,dx$ $= \int e^{6x} + 2e^{3x} + 1\,dx$

$= \boxed{\frac{1}{6}e^{6x} + \frac{2}{3}e^{3x} + x + C}$

(j) $\int x^e\,dx$

$= \boxed{\frac{x^{e+1}}{e+1} + C}$