Name: Solutions

Directions: Show all work. No credit for answers without work.

1. [**4 parts, 3 points each**] The temperature T in degrees Fahrenheit of a frozen pizza placed in a hot oven is given by $T = f(t)$, where t is the time in minutes since the pizza was put in the oven.

   (a) What is the sign of $f'(t)$? Briefly explain your answer.

   Positive, since temperature is increasing

   (b) What are the units of $f'(t)$?

   Degrees Fahrenheit per minute

   (c) What is the sign of $f''(t)$? Briefly explain your answer.

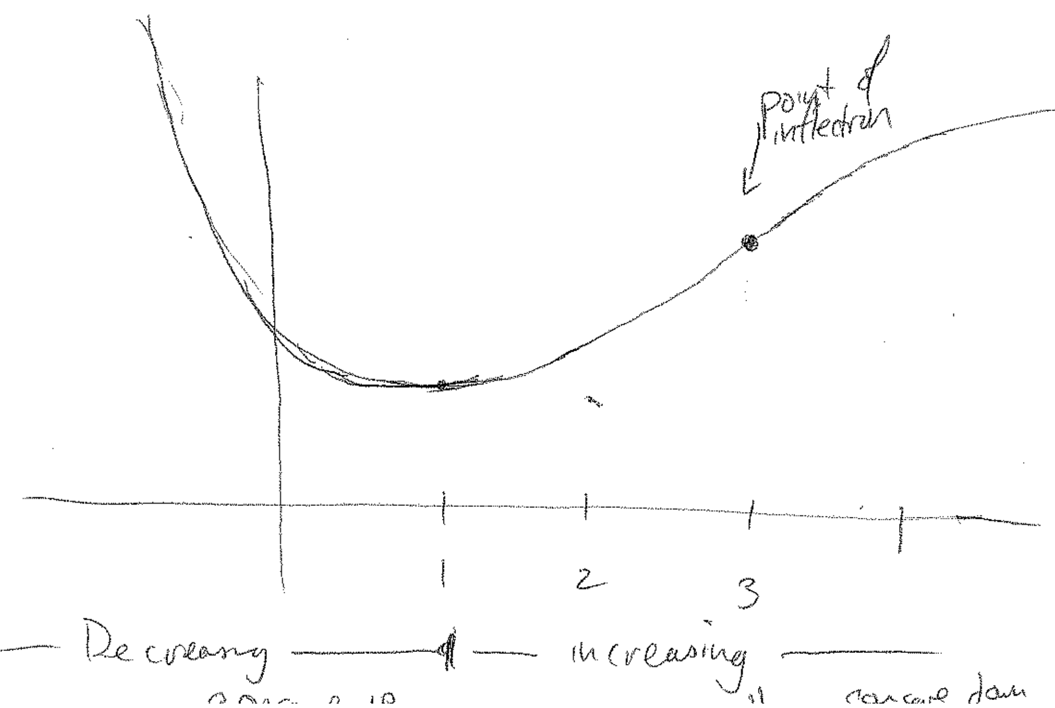
   Negative, since the rate of temperature increase goes down with time

   (d) What are the units of $f''(t)$?

   $\boxed{{}^\circ F / min^2}$

2. [**8 points**] Sketch a graph of a continuous function f with the following properties:

   - When $x < 1$, $f'(x) < 0$; $f'(1) = 0$; and when $x > 1$, $f'(x) > 0$.
   - When $x < 3$, $f''(x) > 0$; $f''(3) = 0$; and when $x > 3$, $f''(x) < 0$.



3. [**10 parts, 2 points each**] Differentiate the following functions.

(a) $f(x) = 4$

0

(b) $f(x) = 3x^2 - 4x + 1$

$6x - 4$

(c) $f(x) = \frac{3}{x^4}$

$-12x^{-5}$

(d) $f(x) = e^{-x}$

$-e^{-x}$

(e) $f(x) = 7^x$

$\ln(7) \cdot 7^x$

(f) $f(x) = 3\sqrt{x}$

$\frac{3}{2}x^{-\frac{1}{2}}$

(g) $f(x) = \ln\left(\sqrt{3} + e^2\right)$

const

0

(h) $f(x) = e^{\sqrt{2} \cdot x}$

$\sqrt{2}\, e^{\sqrt{2}x}$

(i) $f(x) = x^{\ln(4)}$

$\ln(4) \cdot x^{\ln(4) - 1}$

(j) $f(x) = 2\ln(x)$

$\frac{2}{x}$

4. [**4 parts, 5 points each**] Differentiate the following functions.

(a) $f(x) = (x^5 + 2x^3 + 2)(x^4 + 1)$

$$f'(x) = \boxed{(5x^4 + 6x^2)(x^4+1) + (x^5 + 2x^3 + 2)\cdot(4x^3)}$$

(b) $f(x) = (e^x + \ln(x))^8$

$$f'(x) = 8(e^x + \ln(x))^7 \cdot \frac{d}{dx}\left[e^x + \ln(x)\right]$$

$$= \boxed{8(e^x + \ln(x))^7 \cdot \left(e^x + \frac{1}{x}\right)}$$

(c) $f(x) = \dfrac{x^4 + x}{x^2 + 1}$

$$f'(x) = \boxed{\frac{(x^2+1)\cdot(4x^3+1) - (x^4+x)\cdot(2x)}{(x^2+1)^2}}$$

(d) $f(x) = \sqrt{e^{(x^2)} + 1}$

$$f'(x) = \frac{1}{2}\left(e^{(x^2)} + 1\right)^{-\frac{1}{2}} \cdot \frac{d}{dx}\left[e^{(x^2)} + 1\right]$$

$$= \boxed{\frac{1}{2}\left(e^{(x^2)} + 1\right)^{-\frac{1}{2}} \cdot \left(e^{(x^2)} \cdot 2x\right)}$$

$$= \frac{xe^{x^2}}{\sqrt{e^{x^2}+1}}$$

5. Let $g(x) = \ln(x^3 + 1)$.

(a) [**5 points**] Find $g'(x)$.

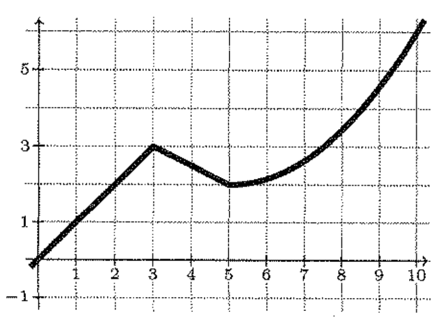
$$g'(x) = \frac{1}{x^3+1} \cdot 3x^2 = \frac{3x^2}{x^3+1}$$

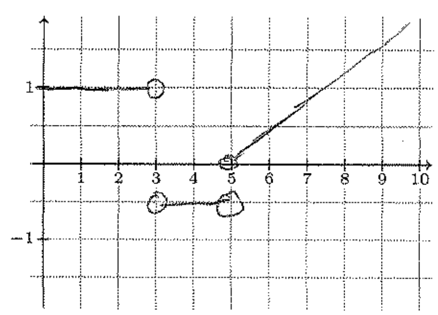
(b) [**5 points**] Find the equation of the tangent line to $g(x)$ at $x = 2$.

- $y - y_0 = m(x - x_0)$
- $(x_0, y_0) = (2, \ln(2^3+1)) = (2, \ln(9))$
- $m = g'(2) = \frac{3 \cdot 2^2}{2^3+1} = \frac{3 \cdot 4}{9} = \frac{4}{3}$
- $y - \ln(9) = \frac{4}{3}(x-2)$

$$\boxed{y = \frac{4}{3}x - \frac{8}{3} + \ln(9)}$$

6. [**10 points**] The graph of $f(x)$ appears below. Sketch $f'(x)$ in the space provided.





7. Let $f(x) = (2x+1)^3(3x+1)$.

(a) [**6 points**] Find $f'(x)$.

$$f'(x) = 3(2x+1)^2 \cdot 2 \cdot (3x+1) + (2x+1)^3 \cdot 3$$

$$= 3(2x+1)^2 \left[2(3x+1) + (2x+1) \right]$$

$$= 3(2x+1)^2 (8x+3)$$

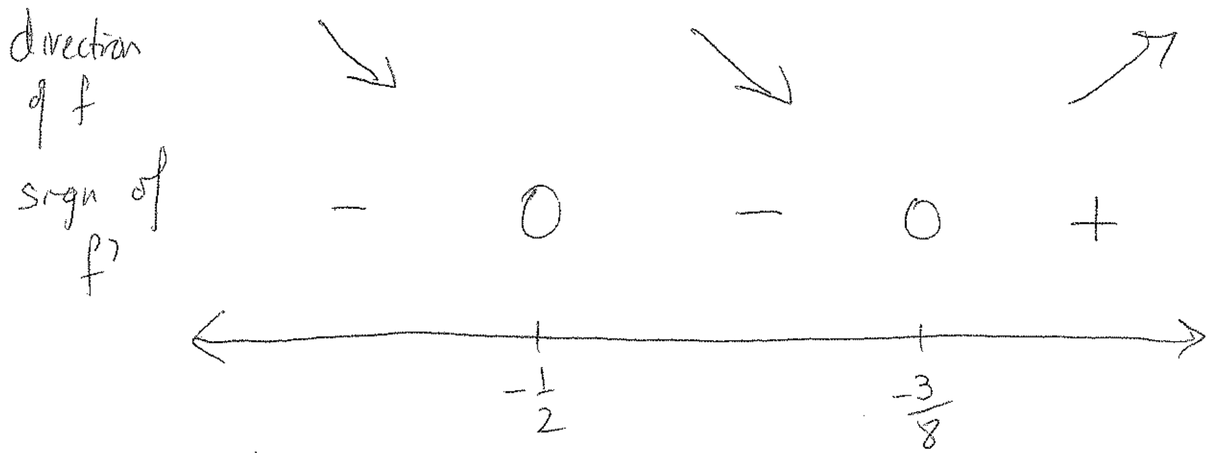
(b) [**7 points**] Find the critical points of f.

$3(2x+1)^2(8x+3) = 0$

$2x+1 = 0$ or $8x+3=0$

$x = -\frac{1}{2}$ or $x = \frac{-3}{8}$

(c) [**7 points**] Use the First Derivative Test to classify each critical point as a local minimum, a local maximum, or neither.



$x = -\frac{1}{2}$: neither

$x = -\frac{3}{8}$: local minimum