Name: ___Solutions___

Directions: Show all work. No credit for answers without work.

1. [**4 parts, 4 points each**] Solve the following for x <u>exactly</u>. Decimal approximations are worth partial credit.

(a) Solve for x in $5^{4x} = 7$.

$$\ln\left(5^{4x}\right) = \ln(7)$$

$$4x \ln(5) = \ln(7)$$

$$\boxed{x = \frac{\ln(7)}{4\ln(5)}}$$

(b) Solve for x in $3\ln(5 - 2x) = 7$.

$$\ln(5 - 2x) = \frac{7}{3}$$

$$e^{\ln(5-2x)} = e^{7/3}$$

$$5 - 2x = e^{7/3}$$

$$\boxed{x = \frac{5 - e^{7/3}}{2}}$$

(c) Let $f(x) = (x - 2)^2$ and $g(x) = -3x + 1$. Find $f(g(-2))$.

$$\circ \; g(-2) = -3(-2) + 1 = 7$$

$$f(g(-2)) = f(7) = (7-2)^2 = \boxed{25}$$

(d) Complete: If $f'(x) > 0$ for each x in $[a, b]$, then $f(x)$ is __increasing__ on $[a, b]$.

2. [**4 points**] A table for $h(x)$ appears below. Using the average of the left-hand and right-hand estimates, give an approximation for $h'(2.4)$.

x	2.30	2.35	2.40	2.45
$h(x)$	35.94	29.64	31.45	34.39

LHE: $h'(2.4) \approx \dfrac{31.45 - 29.64}{0.05} \approx 36.2$

RHE: $h'(2.4) \approx \dfrac{34.39 - 31.45}{0.05} \approx 58.8$

Estimate:

$$h'(2.4) \approx \frac{1}{2}(36.2 + 58.8)$$

$$\approx \boxed{47.5}$$

3. A company rents compact cars for $25 a day plus $0.21 per mile driven and rents pickup trucks for $48 a day plus $0.16 per mile driven.

(a) [4 points] Give a formula $C(x)$ for the cost (in dollars) of renting a car for 2 days when x miles are driven.

$$C(x) = 50 + 0.21x$$

2 days

(b) [4 points] Give a formula $T(x)$ for the cost (in dollars) of renting a truck for 2 days when x miles are driven.

$$T(x) = 96 + 0.16x$$

(c) [8 points] How many miles must be driven for the cost of a 2-day car rental and a 2-day truck rental to be the same?

$$50 + 0.21x = 96 + 0.16x$$

$$0.05x = 46$$

$$x = \boxed{920 \text{ miles}}$$

(d) [2 points] What is the marginal cost of driving a mile in the car?

$$\boxed{\$ 0.21 \text{ per mile}}$$

(e) [2 points] What are the fixed costs of a 2-day truck rental?

$$\boxed{\$96}$$

4. [2 parts, 5 points each] Doug needs to have $18,000 worth of savings in 12 years. Bank A offers an interest rate of 2% per year, compounded annually. Bank B offers an interest rate of 1.5% per year, compounded continuously.

(a) If Doug uses Bank A, how much money should he deposit now?

$$P = P_0(1+r)^t$$
$$18 = P_0(1.02)^{12}$$

$$P_0 = \frac{18}{(1.02)^{12}} \approx 14,1929$$

Doug needs $\boxed{\$14,193}$ now.

(b) If Doug uses Bank B, how much money should he deposit now?

$$P = P_0 e^{kt}$$
$$18 = P_0 e^{0.015(12)}$$

$$P_0 = \frac{18}{e^{(0.015)(12)}} \approx 15.035$$

$\boxed{\$15,035}$ now

Doug needs ~~$15,035~~

5. [2 parts, 5 points each] A cancerous growth of 0.10 grams forms in a patient and grows exponentially. After 3 weeks, the growth has reached a mass of 0.14 grams. The growth is not detectable until it reaches a mass of 0.5 grams.

(a) Give a formula $M(t)$ for the mass (in grams) of the growth after t weeks.

$$M = M_0 a^t$$
$$M = (0.1) a^t$$
$$0.14 = (0.1) a^3$$

$$a^3 = 1.4$$
$$a = (1.4)^{1/3}$$

$$M = \boxed{0.1\left(1.4\right)^{t/3}}$$

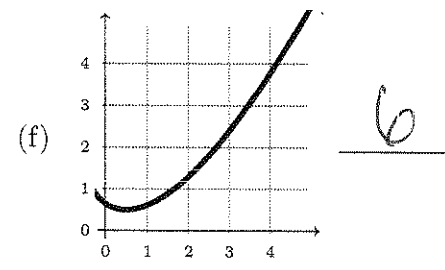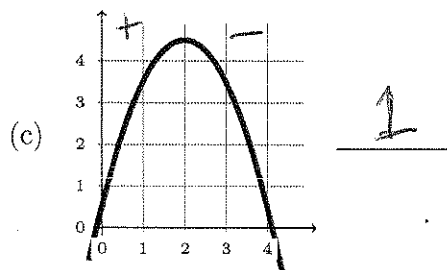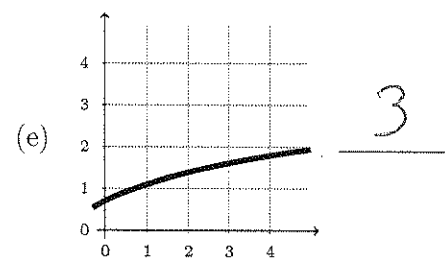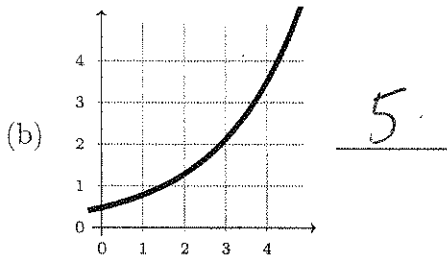(b) How much time will elapse before the growth is detectable?
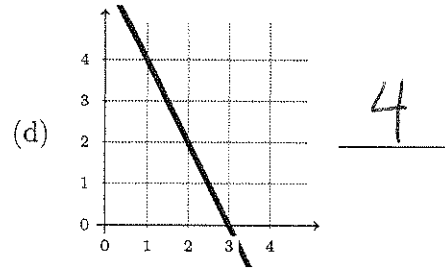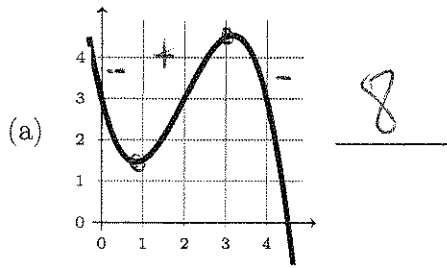
$$0.5 = 0.1\left(1.4\right)^{t/3}$$
$$5 = (1.4)^{t/3}$$
$$\ln(5) = \frac{t}{3}\ln(1.4)$$

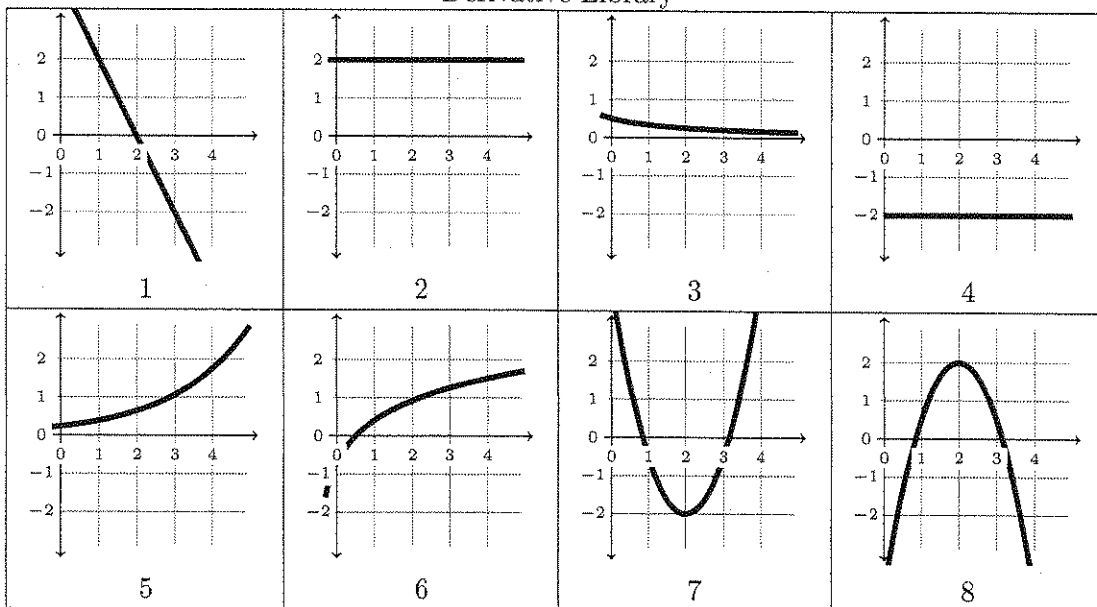$$t = \frac{3\ln(5)}{\ln(1.4)}$$

$$\approx \boxed{14.35 \text{ weeks}}$$

6. **[6 parts, 3 points each]** In each part below, find the derivative of the given graph and write the corresponding number in the provided space. You may use a number more than once. You do not need to show your work.

(a) ___8___

(b) ___5___

(c) ___1___

(d) ___4___

(e) ___3___

(f) ___6___

Derivative Library

| 1 | 2 | 3 | 4 |
| 5 | 6 | 7 | 8 |

7. Let $f(x) = 5x^2$.

(a) [**10 points**] Find the average rate of change of f over the interval $[2, 3]$.

$$\text{ARC} = \frac{f(3) - f(2)}{3 - 2} = \frac{5 \cdot 3^2 - 5 \cdot 2^2}{1} = 5(9 - 4) = \boxed{25}$$

(b) [**10 points**] Find the average rate of change of f over the interval $[x, x + h]$.

$$\text{ARC} = \frac{f(x+h) - f(x)}{(x+h) - x} = \frac{5(x+h)^2 - 5x^2}{h}$$

$$= \frac{5(x^2 + 2xh + h^2) - 5x^2}{h}$$

$$= \frac{5x^2 + 10xh + 5h^2 - 5x^2}{h}$$

$$= \frac{h(10x + 5h)}{h}$$

$$= \boxed{10x + 5h}$$

(c) [**2 points**] Using part (b), find $f'(x)$.

$$f'(x) = \lim_{h \to 0} \text{ARC} \approx 10x + 5(\text{very very small number})$$

$$= \boxed{10x}$$