

Name: Solutions**Directions:** Show all work. No credit for answers without work.

1. [2 parts, 2 points each] A definite integral.

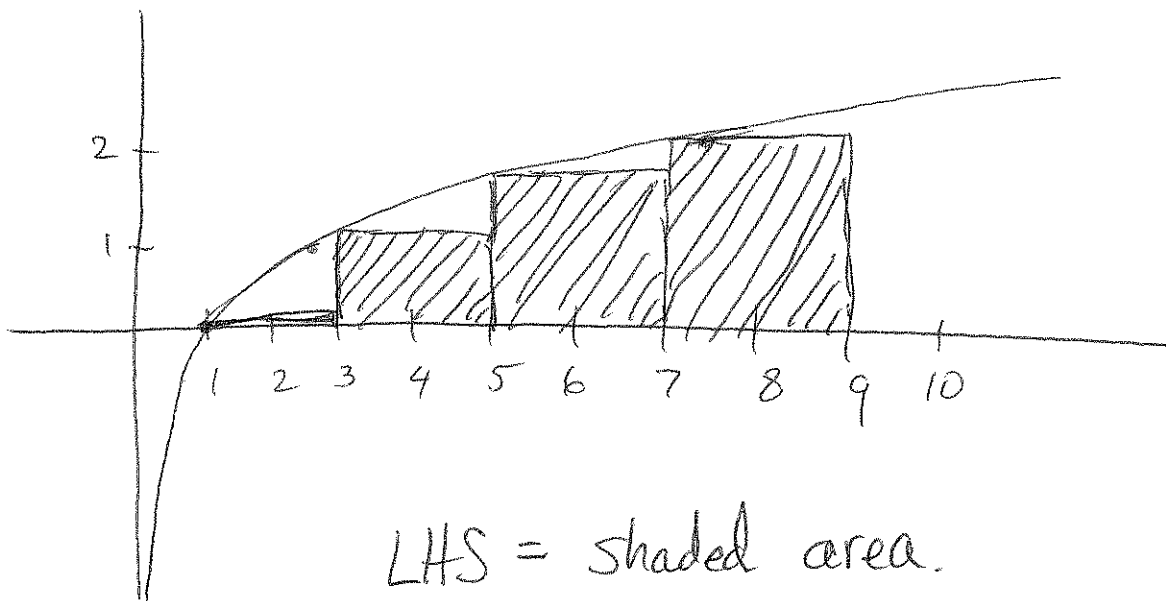
(a) Using $n = 4$, find the left hand sum approximation to $\int_1^9 \ln(x) dx$.

$$\text{LHS: } 2 \cdot 0 + 2 \cdot \ln(3) + 2 \cdot \ln(5) + 2 \cdot \ln(7)$$

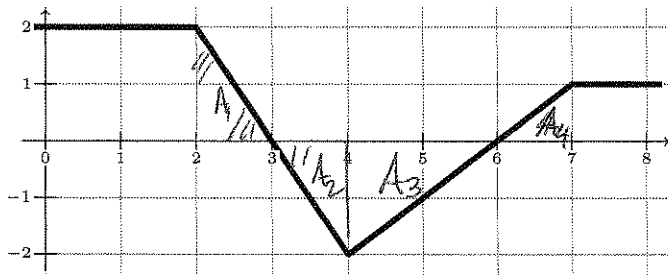
$$\approx 0 + 2.197 + 3.219 + 3.842$$

$$\approx \boxed{9.307}$$

(b) Illustrate your solution to part (a) graphically. Your figure should include a graph of the integrand and the graphical representation of the left hand sum.



2. [2 points] Using the graph of the function $f(x)$ below, find $\int_2^7 f(x) dx$ exactly.



$$\begin{aligned}
 &= A_1 - A_2 - A_3 + A_4 \\
 &= 1 - 1 - \frac{1}{2} \cdot 2 \cdot 2 + \frac{1}{2} \\
 &= 0 - 2 + \frac{1}{2} = \boxed{-1.5}
 \end{aligned}$$

3. [2 points] At time $t = 0$, a large block of ice is removed from a freezer and begins to melt. At time t (in hours), the ice melts at a rate of $2t + 1$ kg per hour. Express the mass of ice that melts during the first 3 hours as a definite integral. (Your answer must be a definite integral; do not solve the integral.)

$$\int_0^3 2t + 1 dt$$

4. [2 parts, 1 point each] The FTC.

(a) State the Fundamental Theorem of Calculus.

If $F'(t)$ is continuous on $[a, b]$, then

$$\int_a^b F'(t) dt = F(b) - F(a).$$

(b) Describe what is represented by each of the two sides of the equation in the Fundamental Theorem of Calculus.

- $\int_a^b F'(t) dt$ is the definite integral of a rate of change; it is the ^{limit of} sum of changes over small intervals of time.
- $F(b) - F(a)$ is the total accumulated change.