Name: Solutions

Directions: Show all work. No credit for answers without work.

1. [**6 points**] Find the derivatives of the following functions.

(a) $y = 4x + 7$

$y' = 4$

(b) $y = x^8$

$y' = 8x^7$

(c) $y = 2x^{2.5}$

$y' = 5x^{1.5}$

(d) $y = 2x^9 - 4x^3$

$y' = 18x^8 - 12x^2$

(e) $y = \sqrt{x} = x^{\frac{1}{2}}$

$y' = \boxed{\frac{1}{2}x^{-\frac{1}{2}}}$

$= \boxed{\frac{1}{2x^{\frac{1}{2}}}} = \boxed{\frac{1}{2\sqrt{x}}} = \boxed{\frac{\sqrt{x}}{2x}}$

(f) $y = \frac{1}{x^2} = x^{-2}$

$y' = \boxed{-2x^{-3}} = \boxed{\frac{-2}{x^3}}$

(g) $\sqrt{5}x^{\ln 3}$

$y' = \boxed{\sqrt{5} \cdot \ln(3)\, x^{\ln(3) - 1}}$

(h) $y = e^{6x}$

$y' = \boxed{6e^{6x}}$

(i) $y = 7^x$

$y' = \boxed{\ln(7) \cdot 7^x}$

(j) $y = (\ln 9)^x$

$y' = \boxed{\ln(\ln(9)) \cdot (\ln 9)^x}$

(k) $y = 3\ln x$

$\boxed{y' = \frac{3}{x}}$

(l) $y = e^{-x} - \ln(x^2) + 1$ $= e^{-x} - 2\ln(x) + 1$

$y' = -e^{-x} - \frac{2}{x} + 0$

$= \boxed{-e^{-x} - \frac{2}{x}}$

2. **[2 points]** Let $C(q)$ be the total cost (in dollars) of producing q items. Suppose that $C(820) = 2180$ and $C'(820) = 21$.

(a) Estimate the cost of producing 823 items.

$C(823) \approx C(820) + C'(820)\cdot\Delta q$

$\approx 2180 + 21\cdot 3 = \boxed{\$2243}$

(b) Estimate the cost of producing 818 items.

$C(818) \approx 2180 + 21(-2) \approx \boxed{\$2138}$

3. **[2 points]** Find the equation of the line tangent to the graph of $f(x) = 5x + x^2$ at $x = 2$.

Use point-slope.

Point: $x_0 = 2$

$y_0 = f(2)$

$= 5\cdot 2 + 2^2$

$= 14$

Slope:

$f'(x) = 5 + 2x$

$m = f'(2) = 5 + 2\cdot 2$

$= 9$

$y - y_0 = m(x - x_0)$

$\boxed{y - 14 = 9(x - 2)}$

$\boxed{y = 9x - 4}$