3. [3 points] A publishing company offers you two payment options for your book. Plan A calls for an immediate payment of $85,000 and a payment of $10,000 in 2 years, when the book will be complete. Plan B calls for three equal payments of $30,000: one immediately, one after 1 year, and a final payment after 2 years. Assume that money earns interest at a continuous annual rate of 6%.
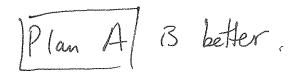
(a) Find the future value of Plan A and Plan B.

Units : P in thousands, t in years

A: $P = 85e^{0.06(2)} + 10$

$t=0$

$= 105.837$, so $\boxed{\$105,837}$

B: $P = 30e^{0.06(2)} + 30e^{0.06(1)} + 30$

$= 95.680$, so $\boxed{\$95,680}$

(b) Find the present value of Plan A and Plan B.

A: $P = P_0 e^{kt}$

$105.837 = P_0 e^{0.06(2)}$

$P_0 = \dfrac{105.837}{e^{0.12}} = 93.869$, so $\boxed{\$93,869}$

B: $P_0 = \dfrac{95.680}{e^{0.12}} = 84.861$, so $\boxed{\$84,861}$

(c) Which plan should you take?

$\boxed{\text{Plan A}}$ is better.

Name: _____Solution_____

Directions: Show all work. No credit for answers without work.

1. [**3 points**] The population of a town decreases exponentially with a discrete annual rate of 1.7%. Find the half-life of the population.

$$P = P_0(1+r)^t \quad , \quad r = -0.017$$

$$P = P_0(1-0.017)^t$$

$$P = P_0(0.983)^t$$

$$\frac{1}{2} = 1(0.983)^t$$

$$\ln\left(\tfrac{1}{2}\right) = t\,\ln(0.983)$$

$$t = \frac{\ln\left(\tfrac{1}{2}\right)}{\ln(0.983)}$$

$$\approx \boxed{40.43 \text{ years}}$$

2. [**4 parts, 1 point each**] Solve the following equations for t <u>exactly</u>. Decimal approximations are worth partial credit.

(a) $5 \cdot 3^t = 12$

$$\ln(5 \cdot 3^t) = \ln(12)$$

$$\ln(5) + t\,\ln(3) = \ln(12)$$

$$\boxed{t = \frac{\ln(12) - \ln(5)}{\ln(3)}}$$

(b) $5 \cdot t^3 = 12$

$$t^3 = \frac{12}{5}$$

$$\boxed{t = \left(\frac{12}{5}\right)^{1/3}}$$

(c) $4\ln(3t+2) = 7$

$$\ln(3t+2) = \frac{7}{4}$$

$$e^{\ln(3t+2)} = e^{7/4}$$

$$3t+2 = e^{7/4}$$

$$3t = e^{7/4} - 2$$

$$\boxed{t = \tfrac{1}{3}\left(e^{7/4} - 2\right)}$$

(d) $8 \cdot 2^t = 5e^{6t}$

$$\ln(8 \cdot 2^t) = \ln(5e^{6t})$$

$$\ln(8) + t\,\ln(2) = \ln(5) + 6t$$

$$\ln(8) - \ln(5) = 6t - t\,\ln(2)$$

$$\ln(8) - \ln(5) = t(6 - \ln(2))$$

$$\boxed{t = \frac{\ln(8) - \ln(5)}{6 - \ln(2)}}$$