

Name: Solution.**Directions:** Show all work. No credit for answers without work.

1. [2 parts, 2 points each] The amount of water W (in millions of gallons) in a lake after t years since 2000 is given by $W(t) = \frac{2600}{t^2+1}$. In both answers below, be sure to include proper units.

- (a) Find the average rate of change in the amount of water between 2002 and 2005.

$$\begin{aligned} \text{ARC} &= \frac{W(5) - W(2)}{5 - 2} \\ &= \frac{\frac{2600}{25+1} - \frac{2600}{4+1}}{3} \end{aligned} \quad \left| \quad \begin{aligned} &= \frac{100 - 520}{3} \\ &= \frac{-420}{3} = \boxed{-140 \text{ million gallons/year}} \end{aligned}$$

- (b) Find the relative change in the amount of water between 2002 and 2005.

$$\begin{aligned} \text{RC} &= \frac{W(5) - W(2)}{W(2)} = \frac{100 - 520}{520} = -\frac{420}{520} \approx \boxed{-0.808} \\ &\text{or } \boxed{-80.8\%} \end{aligned}$$

2. A manufacturer makes glass cups. The total cost C (in dollars) to produce q cups is given by the cost function $C(q) = 127000 + 2q$. Each cup sells for 9 dollars.

- (a) [1 point] What is the manufacturer's marginal cost?

\$2 per cup

- (b) [2 points] How many cups must the manufacturer sell to earn a profit?

$$127000 + 2q = 9q$$

$$127000 = 7q$$

$$q \approx 18142.86$$

So, they must
sell $\boxed{18,143}$ cups.

3. In 2012, the population of a town is 8.3 million. Analysts expect the town's population to shrink by 6.4% per year.

(a) [2 points] Give a formula for the town's population P in millions of people as a function of the number of years t since 2012.

$$P = P_0(1+r)^t$$

$$P = 8.3(1 - 0.064)^t$$

~~$$P = 8.3(1 - 0.064)^t$$~~

$$P = 8.3(0.936)^t$$

(b) [1 point] How many people are expected to live in the town at the beginning of 2019?

• Beginning of 2012: $t=0$.

• " " 2013: $t=1$.

• " " 2019: $t=2019-2012=7$

$$P = 8.3(0.936)^7 = \boxed{5.22 \text{ million people}}$$