

Solutions

HW4: Focus on Theory

(1)

1. (a)

$$\text{ARC} = \frac{f(4) - f(3)}{4 - 3} = \frac{5 \cdot 4^2 - 5 \cdot 3^2}{1}$$

$$= 5 \cdot 16 - 5 \cdot 9$$

Note: This makes
it much easier to
multiply

$$\rightarrow = 5(16 - 9)$$

$$= 5 \cdot 7$$

$$= \boxed{35}$$

$$(b) \text{ ARC} = \frac{f(3+h) - f(3)}{(3+h) - 3} = \frac{5 \cdot (3+h)^2 - 5 \cdot 3^2}{h}$$

$$= \frac{5 \cdot (9 + 6h + h^2) - 5 \cdot 3^2}{h}$$

Again, we
factor out the
5

$$\rightarrow = \frac{5(9 + 6h + h^2 - 3^2)}{h}$$

The +9 cancels
with -3².

$$\rightarrow = \frac{5(6h + h^2)}{h}$$

(2)

Factor out h
from $6h + h^2$ to
get $h(6+h)$.

$$\rightsquigarrow = \frac{5h(6+h)}{h}$$

The h 's cancel $\rightsquigarrow = 5(6+h)$

$$= \boxed{30 + 5h}$$

Note: when $h=1$, we get 35, our answer in part (a).

(c) As $h \rightarrow 0$, ~~from~~ $5h$ gets closer and closer to 0, so $30 + 5h$ gets closer and closer to 30.

$$(d) (\text{IRC of } f \text{ at } x=3) = \lim_{h \rightarrow 0} \text{of ARC of } [3, 3+h]$$

$$= \boxed{30}$$

$$2. (a) \text{ ARC} = \frac{f(1) - f(-1)}{1 - (-1)} = \frac{2 \cdot 1^2 - 2 \cdot (-1)^2}{1 + 1}$$

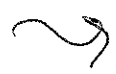
$$= \frac{2 - 2 \cdot 1}{2}$$

$$= \frac{0}{2} = \boxed{0}$$

$$(b) \text{ ARC} = \frac{f(x+h) - f(x)}{(x+h) - x} = \frac{2(x+h)^2 - 2x^2}{h}$$

$$= \frac{2(x^2 + 2xh + h^2) - 2x^2}{h}$$

Factor out
2



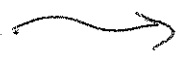
$$= \frac{2((x^2 + 2xh + h^2) - x^2)}{h}$$

x^2 and $-x^2$
cancel



$$= \frac{2(2xh + h^2)}{h}$$

Factor out
h from $2xh + h^2$
to get $h(2x+h)$



$$= \frac{2h(2x+h)}{h}$$

(4)

The h 's
cancel

$$\rightsquigarrow = 2(2x+h)$$

$$= \boxed{4x + 2h}$$

Note: when $x = -1$ and $h = 2$, we get our answer in (a).

(c) As $h \rightarrow 0$, $2h \rightarrow 0$. So $4x + 2h$ gets closer and closer to $4x$.

$$(d) f'(x) = \text{limit of ARC over } [x, x+h] \text{ as } h \rightarrow 0$$
$$= \boxed{4x}$$