Solutions   HW4: Focus on Theory

1. (a)

$$ARC = \frac{f(4)-f(3)}{4-3} = \frac{5\cdot 4^2 - 5\cdot 3^2}{1}$$

$$= 5\cdot 16 - 5\cdot 9$$

Note: This makes it much easier to multiply

$$= 5(16-9)$$

$$= 5\cdot 7$$

$$= \boxed{35}$$

(b) $$ARC = \frac{f(3+h)-f(3)}{(3+h)-3} = \frac{5\cdot(3+h)^2 - 5\cdot 3^2}{h}$$

$$= \frac{5\cdot(9+6h+h^2) - 5\cdot 3^2}{h}$$

Again, we factor out the 5

$$= \frac{5\,(\cancel{9}+6h+h^2-\cancel{3^2})}{h}$$

The +9 cancels with -3^2.

$$= \frac{5(6h+h^2)}{h}$$

Factor out h from $6h + h^2$ to get $h(6+h)$. $\rightsquigarrow$ $= \dfrac{5h(6+h)}{h}$

The h's cancel $\rightsquigarrow$ $= 5(6+h)$

$= \boxed{30 + 5h}$

Note: when $h = 1$, we get 35, our answer in part (a).

(c) As $h \to 0$, 5h gets closer and closer to 0, so $30 + 5h$ gets closer and closer to 30.

(d) $\left(\text{IRC of } f \text{ at } x=3\right) =$ limit of ARC of $[3, 3+h]$ as $h \to 0$

$= \boxed{30}$

2. (a) $$\text{ARC} = \frac{f(1) - f(-1)}{1 - (-1)} = \frac{2\cdot 1^2 - 2\cdot(-1)^2}{1+1}$$

$$= \frac{2 - 2\cdot 1}{2}$$

$$= \frac{0}{2} = \boxed{0}$$

(b) $$\text{ARC} = \frac{f(x+h) - f(x)}{(x+h) - x} = \frac{2(x+h)^2 - 2x^2}{h}$$

$$= \frac{2(x^2 + 2xh + h^2) - 2x^2}{h}$$

factor out 2 $\rightsquigarrow$ $$= \frac{2((x^2 + 2xh + h^2) - x^2)}{h}$$

x^2 and $-x^2$ cancel $\rightsquigarrow$ $$= \frac{2(2xh + h^2)}{h}$$

Factor out h from $2xh + h^2$ to get $h(2x+h)$ $\rightsquigarrow$ $$= \frac{2h(2x+h)}{h}$$

The h's cancel $\rightsquigarrow \; = 2(2x+h)$

$$= \boxed{4x+2h}$$

Note: when $x=-1$ and $h=2$, we get our answer in (a).

(c) As $h \to 0$, $2h \to 0$. So $4x+2h$ gets claer and closer to $4x$.

(d) $f'(x) =$ limit of ARC over $[x, x+h]$ as $h \to 0$

$$= \boxed{4x}$$