$\qquad$

1. A graph of the total cost function $C(q)$ (in thousands of dollars) appears below.

(a) [3 points] Estimate the production level that minimizes marginal cost.
(b) [3 points] Estimate the production level that minimizes average cost.
2. [7 points] The cost of producing $q$ units is given by $C(q)=9 q^{3}-225 q^{2}+6875 q$. Find the production level that minimizes average cost exactly.
3. [7 points] Use the graph of $f(t)$ to estimate the value of the integral $\int_{-2}^{3} f(t) d t$.

4. [8 parts, 3 points each] Evaluate the following indefinite integrals.
(a) $\int 6 d x$
(e) $\int \frac{1}{x^{8}} d x$
(b) $\int z-3 z^{2} d z$
(f) $\int r^{11+\sqrt{2}} d r$
(c) $\int 2 x^{6}(3 x+1) d x$
(g) $\int e^{3} x d x$
(d) $\int e^{7 t} d t$
(h) $\int x^{-1} d x$
5. [2 parts, 5 points each] Evaluate the following indefinite integrals.
(a) $\int \frac{4 x^{3}+3}{\left(x^{4}+3 x+8\right)^{5}} d x$
(b) $\int \frac{e^{\sqrt{x}}}{\sqrt{x}} d x$
6. [6 points] Find the average value of the function $f(x)=x(4-x)$ over the interval [0, 4] exactly.
7. [4 parts, 5 points each] Use the Fundamental Theorem of Calculus to solve the following definite integrals exactly.
(a) $\int_{-2}^{1} 3 x^{2} d x$
(c) $\int_{2}^{5} \frac{(\ln x)^{2}}{x} d x$
(b) $\int_{2}^{4} t^{3}-e^{2 t} d t$
(d) $\int_{0}^{1}\left(x+e^{2 x}\right)\left(x^{2}+e^{2 x}\right)^{10} d x$
8. [4 parts, 3 points each] At time $t=0$ hours, the surface of a pond begins to freeze. The rate $R$ (in inches per hour) of growth in ice is a function $R(t)$ of time.

| $t$ | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
| :---: | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| $R(t)$ | 0 | 0.5 | 1 | 2 | 1.5 | 1 | 0.5 | 0.25 | 0.5 |

(a) Express the total change in the thickness of the ice during the first 8 hours as a definite integral.
(b) With $n=4$, find the Left Hand Sum (LHS) approximation to the above integral.
(c) With $n=8$, find the Left Hand Sum (LHS) approximation to the above integral.
(d) Which of these estimates would you expect to be more accurate? Briefly explain.
9. [8 points] The graph of the derivative $f^{\prime}(x)$ is shown below. Fill in the table of values given that $f(0)=4$.


| $x$ | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
| :---: | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :---: |
| $f(x)$ | 4 |  |  |  |  |  |  |  |

