Name: Answer Key

1. [**8 parts, 1 point each**] Evaluate the following indefinite integrals.

(a) $\int 4\,dx$

$\boxed{4x + C}$

(b) $\int 5z - 2z^2\,dz$

$\boxed{\frac{5}{2}z^2 - \frac{2}{3}z^3 + C}$

(c) $\int e^{4t}\,dt$

$\boxed{\frac{1}{4}e^{4t} + C}$

(d) $\int y(y+1)\,dy$

$= \int y^2 + y\,dy$

$= \boxed{\frac{y^3}{3} + \frac{y^2}{2} + C}$

(e) $\int \frac{1}{x}\,dx$

$= \boxed{\ln|x| + C}$

(f) $\int \sqrt{r}\,dr$ $= \int r^{1/2}\,dr$

$= \frac{r^{3/2}}{3/2} + C$

$= \boxed{\frac{2}{3}r^{3/2} + C}$

(g) $\int e^2\,dx$

$= \boxed{e^2 x + C}$

(h) $\int x^{\sqrt{2}}\,dx$

$= \boxed{\frac{x^{\sqrt{2}+1}}{\sqrt{2}+1} + C}$

2. [**2 parts, 3 points each**] Evaluate the following indefinite integrals.

(a) $\int 4x^3 e^{x^4+2}\, dx$

$w = x^4 + 2$

$dw = 4x^3\, dx$

$\int 4x^3 e^{x^4+2} dx = \int e^{x^4+2} \cdot 4x^3 dx$

$= \int e^w \cdot dw$

$= e^w + C$

$= \boxed{e^{x^4+2} + C}$

(b) $\int \frac{2e^x + 8}{e^x + 4x}\, dx = \int \frac{1}{e^x+4x} \cdot (2e^x+8)\, dx$

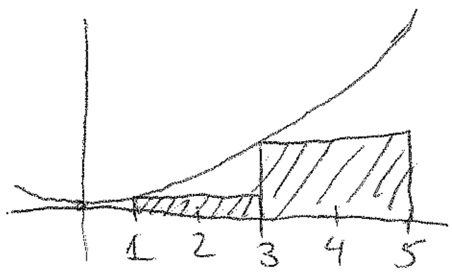
$\boxed{\begin{array}{l} w = e^x + 4x \\ dw = (e^x+4)dx \\ 2dw = (2e^x+8)dx \end{array}}$

$= \int \frac{1}{w} \cdot 2dw$

$= 2 \ln(|w|) + C$

$= \boxed{2\ln(|e^x + 4x|) + C}$

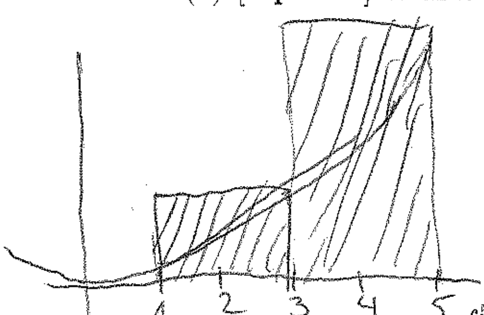
3. (a) [**3 points**] With $n = 2$, give the Left Hand Sum approximation to $\int_1^5 x^2\, dx$.




$LHS = 2 \cdot 1^2 + 2 \cdot 3^2$

$= 2 \cdot 1 + 2 \cdot 9 = 2 + 18 = \boxed{20}$

(b) [**3 points**] With $n = 2$, give the Right Hand Sum approximation to $\int_1^5 x^2\, dx$.

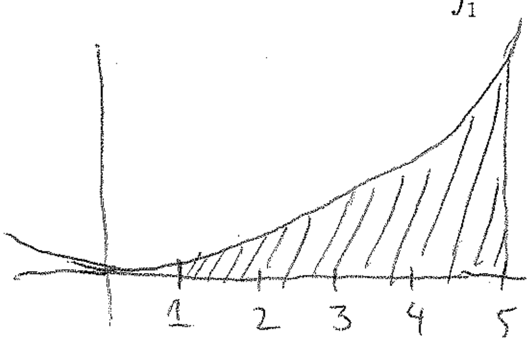



$RHS = 2 \cdot 3^2 + 2 \cdot 5^2$

$= 2 \cdot 9 + 2 \cdot 25$

$= 18 + 50 = \boxed{68}$

(c) [**3 points**] Find $\int_1^5 x^2\, dx$ exactly.




$\int_1^5 x^2 dx = \left.\frac{x^3}{3}\right|_1^5$

$= \frac{5^3}{3} - \frac{1^3}{3} = \frac{125}{3} - \frac{1}{3} = \boxed{\frac{12}{3}}$

4. [**4 parts, 3 points each**] Solve the following definite integrals exactly. Your answers may involve logarithmic and/or exponential functions.

(a) $\int_1^2 4e^{2x}\,dx$

$$= \frac{4}{2} e^{2x}\Big|_1^2$$

$$= 2e^{2x}\Big|_1^2 = \boxed{2e^4 - 2e^2}$$

(b) $\int_4^{25} \frac{1}{\sqrt{t}}\,dt$

$$= \int_4^{25} t^{-1/2}\,dt = 2t^{1/2}\Big|_4^{25}$$

$$= 2\sqrt{25} - 2\sqrt{4}$$

$$= 2\cdot 5 - 2\cdot 2 = 10 - 4 = \boxed{6}$$

(c) $\int_1^e \frac{\ln x}{x}\,dx$

$$= \int_1^e \ln x \cdot \frac{1}{x}\,dx$$

$w = \ln x$

$dw = \frac{1}{x}\,dx$

$$= \int_{\ln(1)}^{\ln(e)} w\,dw$$

$$= \int_0^1 w\,dw = \frac{w^2}{2}\Big|_0^1 = \frac{1^2}{2} - \frac{0^2}{2} = \boxed{\frac{1}{2}}$$

(d) $\int_0^1 (x+2)(x^2+4x)^4\,dx$

$$= \int_0^1 (x^2+4x)^4 \cdot (x+2)\,dx$$

$w = x^2 + 4x$

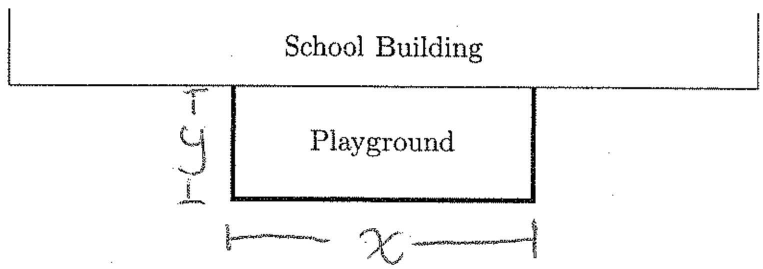
$dw = (2x+4)\,dx$

$\frac{1}{2}\,dw = (x+2)\,dx$

$$= \int_0^5 w^4 \cdot \frac{1}{2}\,dw$$

$$= \frac{w^5}{10}\Big|_0^5 = \frac{5^5}{10} - \frac{0^5}{10} = \boxed{312.5}$$

5. [**14 points**] A community wants to build a playground next to a school. The wall of the school will form the boundary of one side of the playground, and fencing will be used for the other 3 sides (see the figure below). The plans call for the playground to have an area of 72 square meters. What is the minimum amount of fencing needed? Include units and show your work.




$$\text{Area} = xy = 72$$

$$y = \frac{72}{x}$$

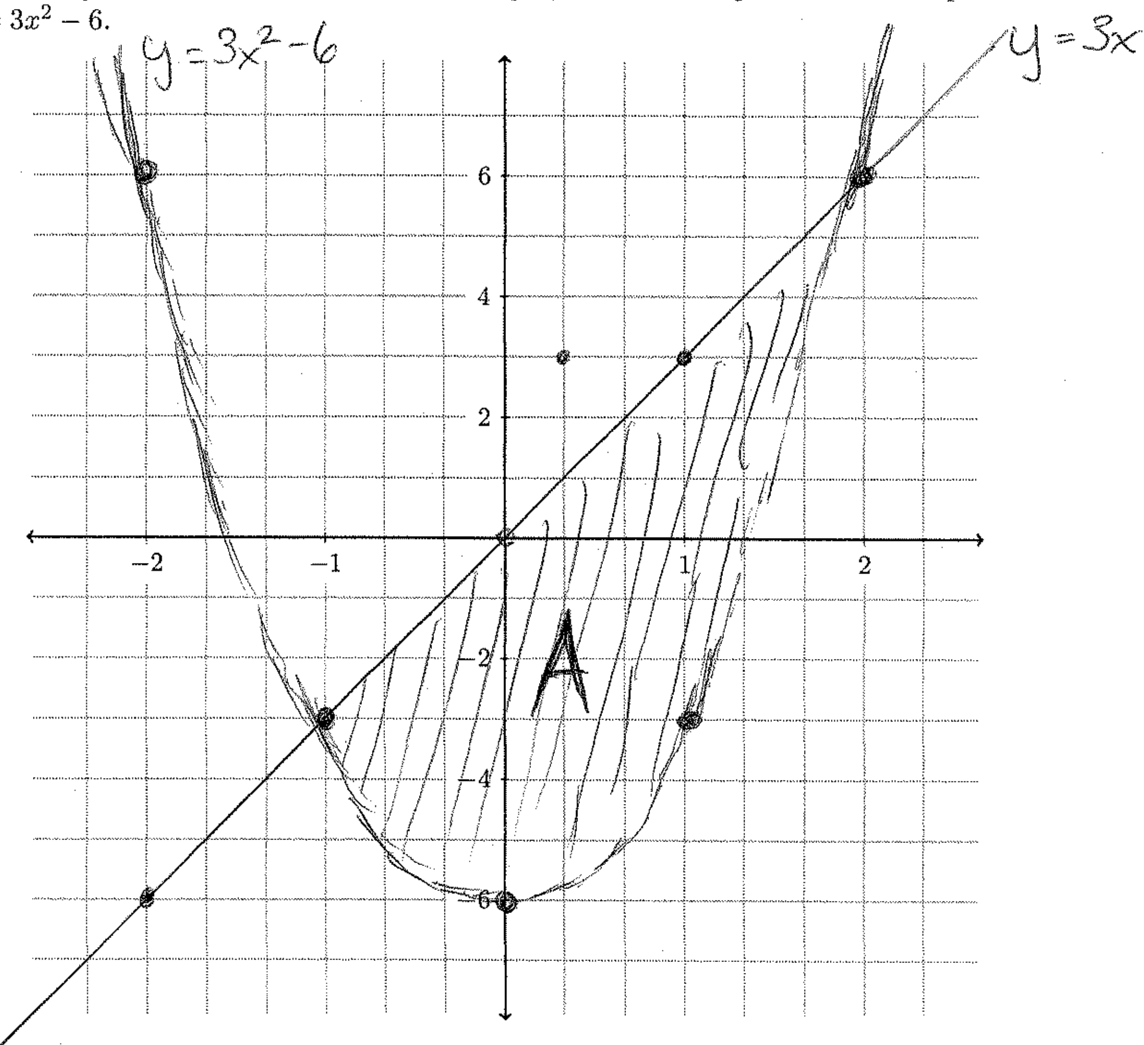
$$\text{Total Fencing} = x + 2y = x + 144/x.$$

- Find the min of $f(x) = x + 144/x$ over $(0, \infty)$
- $f'(x) = 1 + 144(-1)x^{-2}$
  $= 1 - 144/x^2.$
- Crit pts: $1 - \frac{144}{x^2} = 0$
  $1 = \frac{144}{x^2}$
  $x^2 = 144$
  $x = \pm 12$
- Crit points in $(0, \infty)$: $x = 12.$

Check $f(x)$ at $x = 12$ and as $x \to 0$, $x \to \infty$:

- As $x \to 0$, $f(x) \to \infty$.
- As $x \to \infty$, $f(x) \to \infty$
- $f(12) = 12 + 144/12 = 12 + 12$
  $= 24.$
- So the minimum amount of fencing needed is $\boxed{24 \text{ meters}}$.

6. (a) [**4 points**] On the provided grid, sketch a graph of the line $y = 3x$ and the parabola $y = 3x^2 - 6$.




(b) [**6 points**] Express the area of the region bounded by the line $y = 3x$ and the parabola $y = 3x^2 - 6$ as a definite integral. You do not need to solve the integral.

$$A = \boxed{\begin{aligned} &\int_{-1}^{2} (3x) - (3x^2 - 6)\, dx \\ &= \int_{-1}^{2} 3x - 3x^2 + 6 \; dx \end{aligned}}$$

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Do not turn the page until instructed.

**Directions**:

1. Write your name on this page and, after the test begins, on the first page of the test. Writing your names in both locations is worth 1 point.

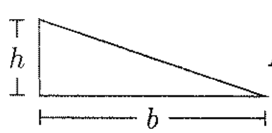
2. Round all numerical answers to three (3) decimal places.

3. Show your work unless you are instructed otherwise. No credit for answers without work.

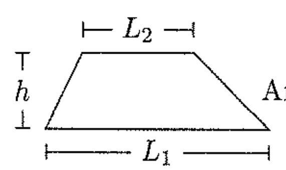
4. You may use a calculator provided it is not equipped with a Computer Algebra System (CAS).

5. Turn off and put away all other electronic equipment (especially cell phones), notes, and books.

6. The test has a total of 60 points. Good luck!




Area $= \frac{1}{2}bh$




Area $= \frac{h}{2}(L_1 + L_2)$

If $ax^2 + bx + c = 0$, then $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.