

Name: \_\_\_\_\_

1. [4 parts, 3 points each] The temperature  $T$  in degrees Fahrenheit of a frozen pizza placed in a hot oven is given by  $T = f(t)$ , where  $t$  is the time in minutes since the pizza was put in the oven.
  - (a) What is the sign of  $f'(t)$ ? Briefly explain your answer.
  
  
  
  
  
  
  
  
  
  
  - (b) What are the units of  $f'(t)$ ?
  
  
  
  
  
  
  
  
  
  
  - (c) What is the sign of  $f''(t)$ ? Briefly explain your answer.
  
  
  
  
  
  
  
  
  
  
  - (d) What are the units of  $f''(t)$ ?
  
2. [8 points] Sketch a graph of a continuous function  $f$  with the following properties:
  - When  $x < 1$ ,  $f'(x) < 0$ ;  $f'(1) = 0$ ; and when  $x > 1$ ,  $f'(x) > 0$ .
  - When  $x < 3$ ,  $f''(x) > 0$ ;  $f''(3) = 0$ ; and when  $x > 3$ ,  $f''(x) < 0$ .

3. [10 parts, 2 points each] Differentiate the following functions.

(a)  $f(x) = 4$

(b)  $f(x) = 3x^2 - 4x + 1$

(c)  $f(x) = \frac{3}{x^4}$

(d)  $f(x) = e^{-x}$

(e)  $f(x) = 7^x$

(f)  $f(x) = 3\sqrt{x}$

(g)  $f(x) = \ln(\sqrt{3} + e^2)$

(h)  $f(x) = e^{\sqrt{2} \cdot x}$

(i)  $f(x) = x^{\ln(4)}$

(j)  $f(x) = 2 \ln(x)$

4. [4 parts, 5 points each] Differentiate the following functions.

(a)  $f(x) = (x^5 + 2x^3 + 2)(x^4 + 1)$

(b)  $f(x) = \frac{x^3}{x+1}$

(c)  $f(x) = (e^x + \ln(x))^8$

(d)  $f(x) = \sqrt{e^{4x} + 1}$

5. Let  $g(x) = (x^2 + 1)^3$ .

(a) [5 points] Find  $g'(x)$ .

(b) [5 points] Find the equation of the tangent line to  $g(x)$  at  $x = -1$ .

6. Mike owns a gas station. The retail price  $R$  (in dollars) that Mike charges his customers for a gallon of gas is given by  $R = \frac{1}{50}B + \frac{1}{3}\ln(B)$ , where  $B$  is the cost (in dollars) of a barrel of crude oil. The cost  $B$  of a barrel of crude oil is, in turn, a function of time  $t$  (in days). Currently, the cost  $B$  of a barrel of crude oil is \$100 and increasing at a rate of \$1.50 per day.

(a) [5 points] Find the current retail price  $R$  of a gallon of gas at Mike's gas station.

(b) [5 points] Find the current rate of change in Mike's retail price in dollars per day.

7. Let  $f(x) = e^x(2x + 1)^4$ .

(a) [**6 points**] Find  $f'(x)$ .

(b) [**7 points**] Find the critical points of  $f$ .

(c) [**7 points**] Use the First Derivative Test or Second Derivative Test to classify each critical point as a local minimum, a local maximum, or neither.