Name: Key

1. **[4 parts, 3 points each]** The temperature T in degrees Fahrenheit of a frozen pizza placed in a hot oven is given by $T = f(t)$, where t is the time in minutes since the pizza was put in the oven.

   (a) What is the sign of $f'(t)$? Briefly explain your answer.

   $f'(t)$ is $\boxed{\text{positive}}$ since the temperature T is increasing

   (b) What are the units of $f'(t)$?

   degrees F / minute

   (c) What is the sign of $f''(t)$? Briefly explain your answer.

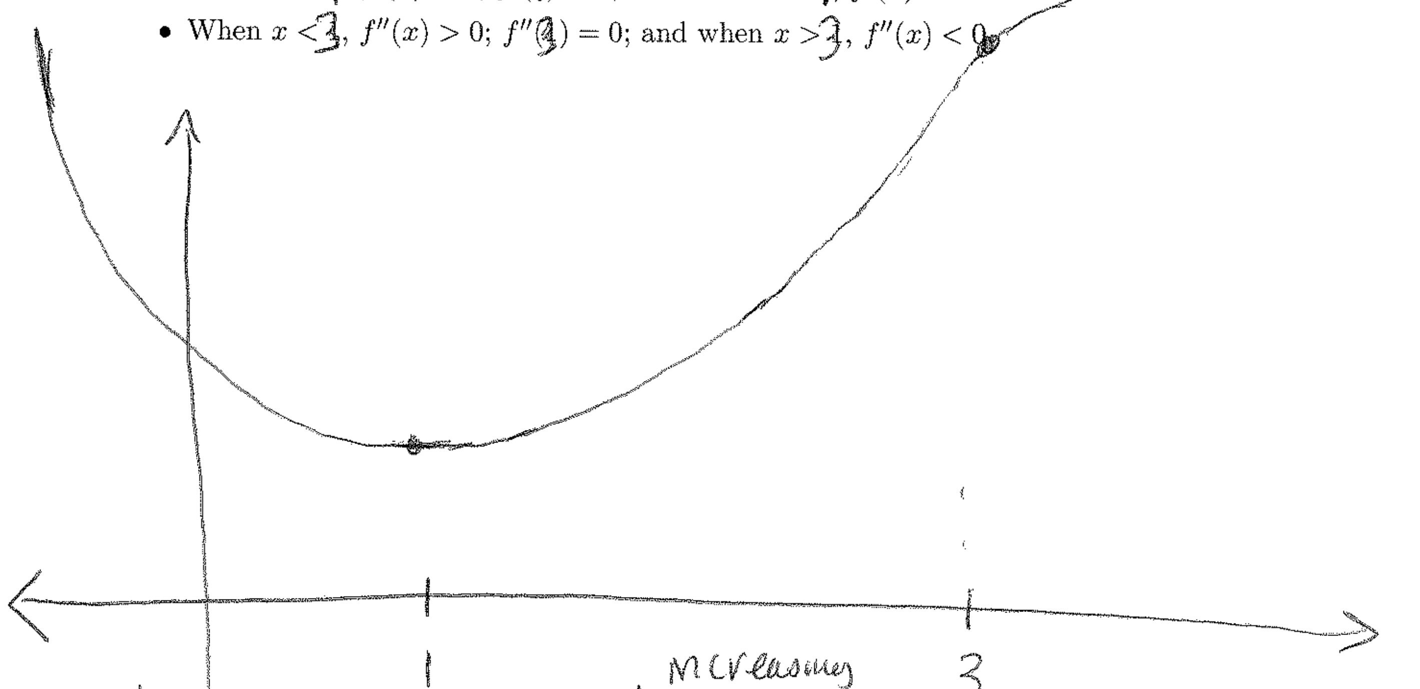
   $f''(t)$ is $\boxed{\text{negative}}$, since the temperature increases more slowly the longer the pizza is in the oven

   (d) What are the units of $f''(t)$?

   (degrees F per minute) per minute

2. **[8 points]** Sketch a graph of a continuous function f with the following properties:

   - When $x < 1$, $f'(x) < 0$; $f'(1) = 0$; and when $x > 1$, $f'(x) > 0$.
   - When $x < 3$, $f''(x) > 0$; $f''(3) = 0$; and when $x > 3$, $f''(x) < 0$.



decreasing
concave up

increasing
~~decreasing~~
concave ~~down~~ up

increasing
concave down

3. [**10 parts, 2 points each**] Differentiate the following functions.

(a) $f(x) = 4$

$\frac{d}{dx}[4] = \boxed{0}$

(b) $f(x) = 3x^2 - 4x + 1$

$\boxed{6x - 4}$

(c) $f(x) = \frac{3}{x^4}$ $= 3x^{-4}$

$\boxed{-12x^{-5}}$

(d) $f(x) = e^{-x}$ e^{kx} with $k=-1$

$\boxed{-e^{-x}}$

(e) $f(x) = 7^x$

$\boxed{\ln(7) \cdot 7^x}$

(f) $f(x) = 3\sqrt{x}$ $= 3x^{\frac{1}{2}}$

$\boxed{\frac{3}{2}x^{-\frac{1}{2}}}$

(g) $f(x) = \ln\left(\sqrt{3} + e^2\right)$

constant

$\boxed{0}$

(h) $f(x) = e^{\sqrt{2}\cdot x}$ e^{kx} with $k = \sqrt{2}$

$\boxed{\sqrt{2}\, e^{\sqrt{2}\cdot x}}$

(i) $f(x) = x^{\ln(4)}$

$\boxed{\ln(4) \cdot x^{\ln(4) - 1}}$

(j) $f(x) = 2\ln(x)$

$2 \cdot \frac{1}{x} = \boxed{\frac{2}{x}}$

4. [**4 parts, 5 points each**] Differentiate the following functions.

(a) $f(x) = (x^5 + 2x^3 + 2)(x^4 + 1)$

$$f'(x) = \frac{d}{dx}\left[x^5 + 2x^3 + 2\right](x^4+1) + \cancel{\tfrac{d}{dx}}(x^5+2x^3+2)\frac{d}{dx}\left[x^4+1\right]$$

$$= \boxed{(5x^4+6x^2)(x^4+1) + (x^5+2x^3+2)\cdot 4x^3}$$

(b) $f(x) = \dfrac{x^3}{x+1}$

$$f'(x) = \frac{(x+1)\cdot \frac{d}{dx}\left[x^3\right] - x^3\cdot\frac{d}{dx}\left[x+1\right]}{(x+1)^2}$$

$$= \frac{(x+1)\cdot 3x^2 - x^3}{(x+1)^2} = \boxed{\frac{3x^2(x+1) - x^3}{(x+1)^2}}$$

(c) $f(x) = (e^x + \ln(x))^8$

$$f'(x) = 8\left(e^x + \ln(x)\right)^7 \cdot \frac{d}{dx}\left[e^x + \ln(x)\right]$$

$$= \boxed{8\left(e^x + \ln(x)\right)^7 \cdot \left(e^x + \frac{1}{x}\right)}$$

(d) $f(x) = \sqrt{e^{4x} + 1}$ $= (e^{4x}+1)^{1/2}$

$$f'(x) = \tfrac{1}{2}(e^{4x}+1)^{-\frac{1}{2}} \cdot \frac{d}{dx}\left[e^{4x} + 1\right]$$

$$= \frac{1}{2\sqrt{e^{4x}+1}} \cdot \left(4e^{4x} + 0\right) = \boxed{\frac{2e^{4x}}{\sqrt{e^{4x}+1}}}$$

5. Let $g(x) = (x^2+1)^3$.

(a) [**5 points**] Find $g'(x)$.

$$g'(x) = 3(x^2+1)^2 \cdot \frac{d}{dx}[x^2+1]$$
$$= 3(x^2+1)^2 \cdot (2x+0)$$
$$= 6x(x^2+1)^2$$

(b) [**5 points**] Find the equation of the tangent line to $g(x)$ at $x = -1$.

$$y - y_1 = m(x - x_1)$$

$$y - 8 = -24(x-(-1))$$

$$\boxed{y = -24x - 16}$$

- $x_1 = -1$
- $y_1 = g(-1) = ((-1)^2+1)^3 = (1+1)^3 = 2^3 = 8$
- $m = g'(-1) = 6(-1)((-1)^2+1)^2 = -6(1+1)^2 = -6 \cdot 4 = -24$

6. Mike owns a gas station. The retail price R (in dollars) that Mike charges his customers for a gallon of gas is given by $R = \frac{1}{50}B + \frac{1}{3}\ln(B)$, where B is the cost (in dollars) of a barrel of crude oil. The cost B of a barrel of crude oil is, in turn, a function of time t (in days). Currently, the cost B of a barrel of crude oil is \$100 and increasing at a rate of \$1.50 per day.

(a) [**5 points**] Find the current retail price R of a gallon of gas at Mike's gas station.

$$R = \frac{1}{50} \cdot 100 + \frac{1}{3}\ln(100) \approx 2 + 1.54 = \boxed{\$3.54}$$

(b) [**5 points**] Find the current rate of change in Mike's retail price in dollars per day.

$$\frac{dR}{dt} = \frac{dR}{dB} \cdot \frac{dB}{dt}$$
$$= \frac{d}{dB}\left[\frac{1}{50}B + \frac{1}{3}\ln(B)\right] \cdot \frac{dB}{dt}$$
$$= \left(\frac{1}{50} + \frac{1}{3B}\right) \cdot \frac{dB}{dt}$$

- Plug in $B = 100$, $\frac{dB}{dt} = 1.5$:

$$= \left(\frac{1}{50} + \frac{1}{3 \cdot 100}\right) \cdot 1.5$$
$$= \frac{7}{300} \cdot 1.50$$
$$= \boxed{0.035 \text{ dollars/day}}$$

7. Let $f(x) = e^x(2x+1)^4$.

(a) [**6 points**] Find $f'(x)$.

$$f'(x) = \frac{d}{dx}\left[e^x\right](2x+1)^4 + e^x\frac{d}{dx}\left[(2x+1)^4\right]$$
$$= e^x(2x+1)^4 + e^x\,4(2x+1)^3\cdot\frac{d}{dx}\left[2x+1\right]$$
$$= e^x(2x+1)^4 + e^x\cdot 4\cdot(2x+1)^3\cdot 2$$
$$= e^x(2x+1)^3\left[(2x+1) + 8\right]$$

(b) [**7 points**] Find the critical points of f.

$$e^x(2x+1)^3(2x+9) = 0$$

$e^x = 0$ or $(2x+1)^3 = 0$ or $2x+9=0$

No soln

$2x+1=0$

$x = -\frac{1}{2}$

$2x = -9$

$x = \frac{-9}{2}$

Critical pts:

$x = -\frac{9}{2}, -\frac{1}{2}$

(c) [**7 points**] Use the First Derivative Test or Second Derivative Test to classify each critical point as a local minimum, a local maximum, or neither.

Use FDT.

f: ↗ ↘ ↗

sign f': + 0 − 0 +

$-\frac{9}{2}$ $\quad$ $-\frac{1}{2}$

local max $\quad$ local min