

Name: \_\_\_\_\_

Show your work. Answers without work earn reduced credit. This test has 100 points.

1. [4 parts, 5 points each] Solve the following equations for  $t$  *exactly*. Decimal approximations are worth partial credit.

(a)  $6^{-2t} = 8.$

(c)  $e^{5t} = 2^{t+1}.$

(b)  $9\left(\frac{3}{7}\right)^t = 8.$

(d)  $4\ln(8 - 3t) = 12.$

2. [2 parts, 6 points each] Tables for  $f(x)$  and  $g(x)$  appear below. Each function is either linear or exponential. Give a formula for each function.

(a) 

$x$	4	5	6	7
$f(x)$	5	2	-1	-4

(b) 

$x$	-1	0	1	2
$g(x)$	16	24	36	54

3. A movie theater incurs \$8000 in fixed expenses each day. Each customer costs the theater an additional \$2.00. The theater sells movie tickets for \$10.

(a) [3 points] Give a formula  $C(q)$  for the cost (in dollars) of running the theater for a day when the theater sells  $q$  movie tickets.

(b) [3 points] Give a formula  $R(q)$  for the revenue (in dollars) received on a day when  $q$  tickets are sold.

(c) [6 points] How many tickets must be sold in a day for the theater to break even?

4. In 2000, Town A had a population of 3 million. The population of Town A grows at a discrete rate of 4% each year. Town B had a population of 8.2 million in 2000 and declines at a discrete rate of 2.5% each year.

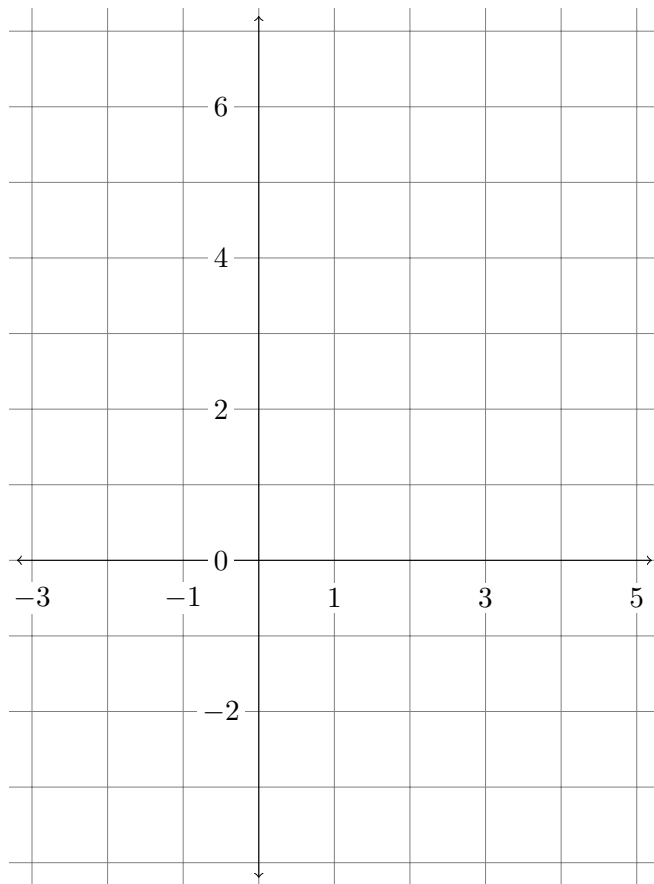
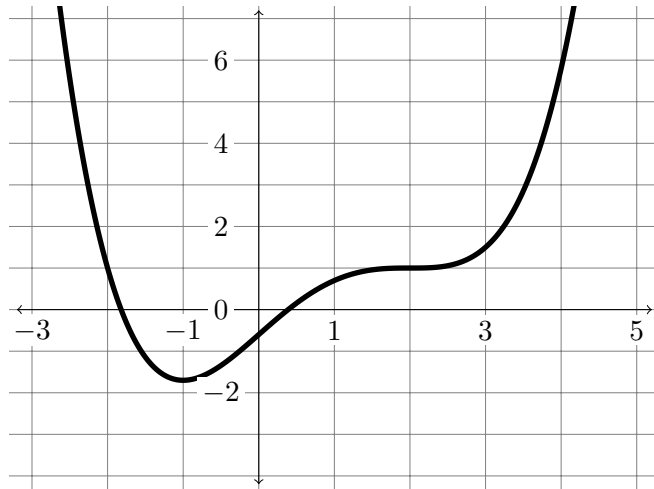
(a) **[3 points]** Find a formula for the population  $P$  (in millions) of Town A.

(b) **[3 points]** Find a formula for the population  $P$  (in millions) of Town B.

(c) **[8 points]** What is the half-life of the population of Town B?

(d) **[8 points]** When will the towns have the same population?

5. [12 points] The graph of a function  $f(x)$  appears below. Sketch the derivative  $f'(x)$ . Your sketch of  $f'(x)$  should capture the important features of  $f'(x)$ .



6. Let  $f(x) = 4x^2$ .

(a) **[6 points]** Find the average rate of change of  $f$  over the interval  $[1, 3]$ .

(b) **[14 points]** Find the average rate of change of  $f$  over the interval  $[x, x + h]$ .

(c) **[2 points]** Using part (b), find  $f'(x)$ .