Name: Key

Show your work. Answers without work earn reduced credit.

1. [**4 parts, ~~4~~ 5 points each**] Solve the following equations for t *exactly*. Decimal approximations are worth partial credit.

(a) $6^{-2t} = 8$.

$$\ln(6^{-2t}) = \ln(8)$$

$$-2t \cdot \ln(6) = \ln(8)$$

$$t = \boxed{\frac{\ln(8)}{-2\ln(6)}}$$

(b) $9\left(\frac{3}{7}\right)^t = 8$.

$$\left(\frac{3}{7}\right)^t = \frac{8}{9}$$

$$\ln\left(\left(\frac{3}{7}\right)^t\right) = \ln\left(\frac{8}{9}\right)$$

$$t \ln\left(\frac{3}{7}\right) = \ln\left(\frac{8}{9}\right)$$

$$t = \boxed{\frac{\ln\left(\frac{8}{9}\right)}{\ln\left(\frac{3}{7}\right)} = \frac{\ln(8) - \ln(9)}{\ln(3) - \ln(7)}}$$

(c) $e^{5t} = 2^{t+1}$.

$$\ln(e^{5t}) = \ln(2^t)$$

$$5t = (t+1)\ln(2)$$

$$5t = t \cdot \ln(2) + \ln(2)$$

$$5t - t \cdot \ln(2) = \ln(2)$$

$$t(5 - \ln(2)) = \ln(2)$$

$$\boxed{t = \frac{\ln(2)}{5 - \ln(2)}}$$

(d) $4\ln(8 - 3t) = 12$.

$$\ln(8 - 3t) = 3$$

$$e^{\ln(8-3t)} = e^3$$

$$8 - 3t = e^3$$

$$-3t = e^3 - 8$$

$$\boxed{t = \frac{e^3 - 8}{-3}}$$

2. **[2 parts, 5 points each]** Tables for $f(x)$ and $g(x)$ appear below. Each function is either linear or exponential. Give a formula for each function.

(a)

x	4	5	6	7
$f(x)$	5	2	-1	-4

Linear. $m = \frac{\Delta y}{\Delta x} = \frac{-3}{1}$

$y - y_0 = m(x - x_0)$

$y - 5 = -3(x-4)$

$\boxed{y = -3x + 17}$

(b)

x	-1 $\cancel{2}$	0 $\cancel{3}$	1 $\cancel{4}$	2 $\cancel{5}$
$g(x)$	16	24	36	54

Exponential: $P = P_0 a^t$

$P_0 = 24$.

$36 = 24 \cdot a^1$

$a = \frac{3}{2}$.

$\boxed{P = 24 \cdot \left(\frac{3}{2}\right)^t}$

3. A movie theater incurs \$8000 in fixed expenses each day. Each customer costs the theater an additional ~~\$3.50~~ 2.00. The theater sells movie tickets for \$10.

(a) **[2 points]** Give a formula $C(q)$ for the cost (in dollars) of running the theater for a day when the theater sells q movie tickets.

$C(q) = 8000 + 2q$

(b) **[2 points]** Give a formula $R(q)$ for the revenue (in dollars) received on a day when q tickets are sold.

$R(q) = 10q$

(c) **[6 points]** How many tickets must be sold in a day for the theater to break even?

$8000 + 2q = 10q$

$8000 = 8q$

$q =$ $\boxed{\text{1000 tickets must be sold.}}$

4. In 2000, Town A had a population of 3 million. The population of Town A grows at a discrete rate of 4% each year. Town B had a population of 8.2 million in 2000 and declines at a discrete rate of 2.5% each year.

(a) [**3 points**] Find a formula for the population P (in millions) of Town A.

$$P = P_0(1+r)^t$$
$$P = 3(1+0.04)^t$$
$$\boxed{P = 3\cdot(1.04)^t}$$

(b) [**3 points**] Find a formula for the population P (in millions) of Town B.

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$$P = 8.2(1-0.025)^t$$
$$P = 8.2(0.975)^t$$

(c) [~~6~~ 7 **points**] What is the half-life of the population of Town B?

$$4.1 = 8.2(0.975)^t$$
$$\tfrac{1}{2} = (0.975)^t$$
$$\ln\left(\tfrac{1}{2}\right) = \ln\left((0.975)^t\right)$$
$$\ln\left(\tfrac{1}{2}\right) = t\cdot\ln(0.975)$$

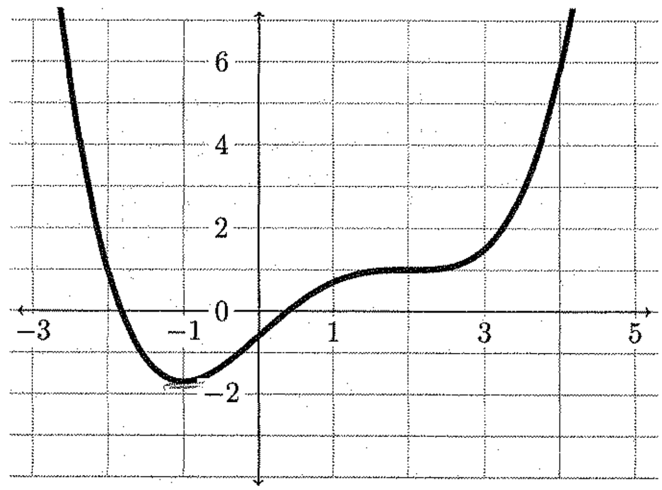
$$t = \boxed{\frac{\ln\left(\frac{1}{2}\right)}{\ln(0.975)}\text{ years} \approx 27.38\text{ years}}$$

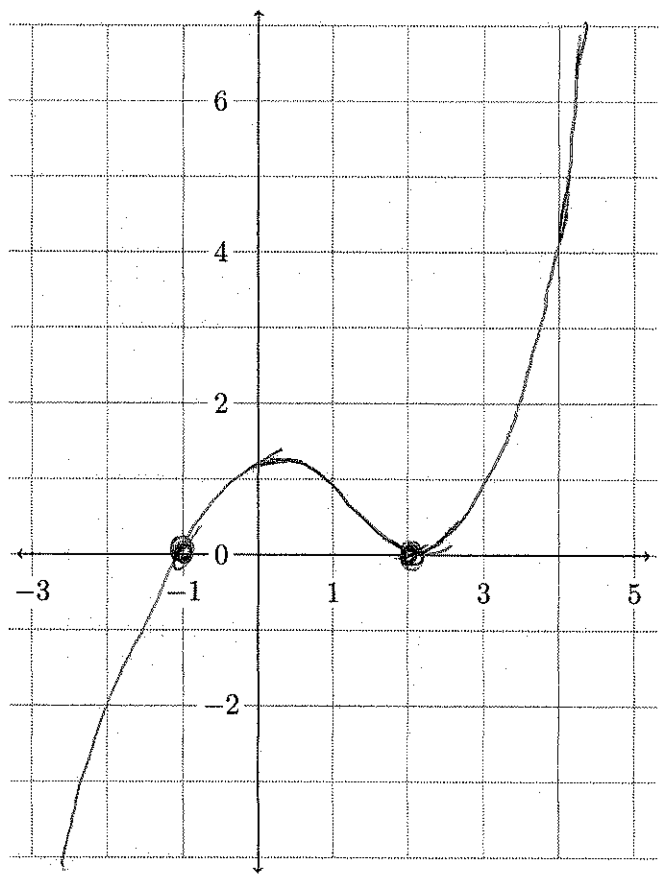
(d) [~~6~~ 7 **points**] When will the towns have the same population?

$$3(1.04)^t = 8.2(0.975)^t$$
$$\ln\left(3\cdot(1.04)^t\right) = \ln\left(8.2\cdot(0.975)^t\right)$$
$$\ln(3) + t\cdot\ln(1.04) = \ln(8.2) + t\ln(0.975)$$
$$t\left(\ln(1.04) - \ln(0.975)\right) = \ln(8.2) - \ln(3)$$
$$t = \frac{\ln(8.2)-\ln(3)}{\ln(1.04)-\ln(0.975)} \approx \boxed{15.58\text{ years}}$$

The towns will have the same pop. in $\boxed{2015}$.

6. [~~15~~ 20 **points**] The graph of a function $f(x)$ appears below. Sketch the derivative $f'(x)$.





7. Let $f(x) = 4x^2$.

(a) 5 [~~4~~ **points**] Find the average rate of change of f over the interval $[1, 3]$.

$$ARC = \frac{f(3)-f(1)}{3-1} = \frac{4\cdot 3^2 - 4}{2} = \frac{36-4}{2}$$

$$= \frac{32}{2} = \boxed{16}$$

(b) 14 [~~10~~ **points**] Find the average rate of change of f over the interval $[x, x+h]$.

$$ARC = \frac{f(x+h) - f(x)}{(x+h) - x} = \frac{4(x+h)^2 - 4x^2}{h}$$

$$= \frac{4(x^2+2xh+h^2) - 4x^2}{h}$$

$$= \frac{4x^2+8xh+4h^2-4x^2}{h}$$

$$= \frac{8xh+4h^2}{h}$$

$$= \frac{\cancel{h}(8x+4h)}{\cancel{h}}$$

$$= \boxed{8x+4h}$$

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(c) [~~1~~ **point**] Using part (b), find $f'(x)$.

$$f'(x) = \lim_{h \to 0} 8x+4h = 8x+4\cdot 0 = \boxed{8x}$$