Name: Answer Key

1. [**4 parts, 2 points each**] Short Answer Questions.

(a) Solve for x in $5^{2x} = 7$.

$\ln(5^{2x}) = \ln(7)$

$2x \ln(5) = \ln(7)$

$2x = \frac{\ln(7)}{\ln(5)}$

$x = \frac{\ln(7)}{2 \cdot \ln(5)} \approx \boxed{0.60}$

(b) Solve for x in $3\ln(5-2x) = 12$.

$\ln(5-2x) = 4$

$5-2x = e^4$

$5-e^4 = 2x$

$x = \frac{5-e^4}{2}$

$x \approx \boxed{-24.80}$

(c) Let $f(x) = (x-2)^2$ and $g(x) = -3x+1$. Find $f(g(-2))$.

$g(-2) = (-3)(-2)+1 = 6+1 = 7$

$f(g(-2)) = f(7) = (7-5)^2 = 5^2 = \boxed{25}$

(d) Complete: If $f'(x) > 0$ for each x in $[a,b]$, then $f(x)$ is increasing on $[a,b]$.

2. [**4 points**] A table for $h(x)$ appears below. Using the average of the left-hand and right-hand estimates, give an approximation for $h'(2.4)$.

x	2.30	2.35	2.40	2.45
$h(x)$	22.94	24.64	26.45	28.39

LH estimate: $\frac{26.45-24.64}{0.05} = \frac{1.81}{0.05} = 36.2$

RH estimate: $\frac{28.39-26.45}{0.05} = \frac{1.94}{0.05} = 38.8$

$f'(2.4) \approx \frac{1}{2}(36.2+38.8) = \boxed{37.5}$

3. A company rents compact cars for \$30 a day plus \$0.18 per mile driven and rents pickup trucks for \$45 a day plus \$0.11 per mile driven.

(a) [**2 points**] Give a formula $C(x)$ for the cost (in dollars) of renting a car for 2 days when x miles are driven.

$C(x) = 60 + 0.18x$

(b) [**2 points**] Give a formula $T(x)$ for the cost (in dollars) of renting a truck for 2 days when x miles are driven.

$T(x) = 90 + 0.11x$

(c) [**4 points**] How many miles must be driven for the cost of a 2-day car rental and a 2-day truck rental to be the same?

$60 + 0.18x = 90 + 0.11x$

$0.07x = 30$

$x = \frac{30}{0.07}$

$x \approx 428.57$ miles

(d) [**1 point**] What is the marginal cost of driving a mile in the car? \$0.18

(e) [**1 point**] What are the fixed costs of a 2-day truck rental? \$90

4. **[2 parts, 3 points each]** Doug needs to have \$20,000 worth of savings in 12 years. Bank A offers an interest rate of 8% per year, compounded annually. Bank B offers an interest rate of 7.7% per year, compounded continuously.

   (a) If Doug uses Bank A, how much money should he deposit now?

$$P(t) = P_0(1+r)^t$$
$$20{,}000 = P_0(1+0.08)^{12}$$
$$P_0 = \frac{20{,}000}{(1.08)^{12}} \approx \boxed{\$7942.28}$$

   (b) If Doug uses Bank B, how much money should he deposit now?

$$P(t) = P_0 e^{kt}$$
$$20000 = P_0 e^{0.077(12)}$$
$$P_0 = \frac{20000}{e^{0.077 \cdot 12}} = \boxed{\$7938.56}$$

5. **[2 parts, 4 points each]** A cancerous growth of 0.10 grams forms in a patient and grows exponentially. After 3 weeks, the growth has reached a mass of 0.14 grams.

   (a) Give a formula $M(t)$ for the mass (in grams) of the growth after t weeks.

$$M(t) = M_0 \cdot e^{kt}$$
$$0.14 = 0.1\, e^{k \cdot 3}$$
$$1.4 = e^{3k}$$

$$\ln(1.4) = 3k$$
$$k = \frac{\ln(1.4)}{3} = 0.112$$

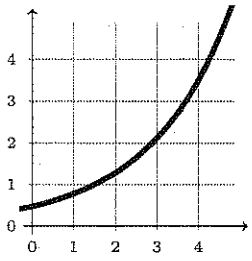
$$\boxed{M(t) = 0.1\, e^{0.112t}}$$

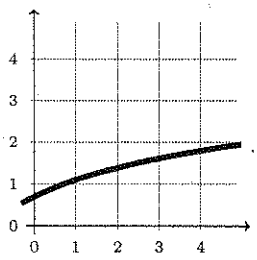
   (b) What is the doubling time of $M(t)$?

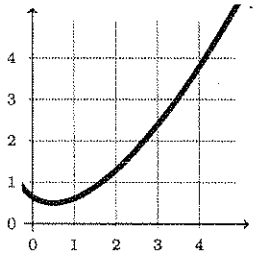
$$0.2 = 0.1\, e^{0.112t}$$
$$2 = e^{0.112t}$$
$$\ln(2) = 0.112t$$

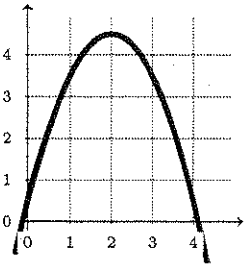
$$t = \frac{\ln(2)}{0.112}$$
$$= \boxed{6.19 \text{ weeks}}$$

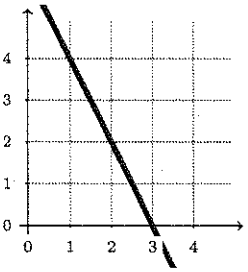
6. [**6 parts, 2 points each**] In each part below, find the derivative of the given graph and write the corresponding number in the provided space. You may use a number more than once. You do not need to show your work.

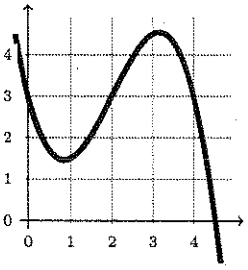
(a)  5

(b)  3

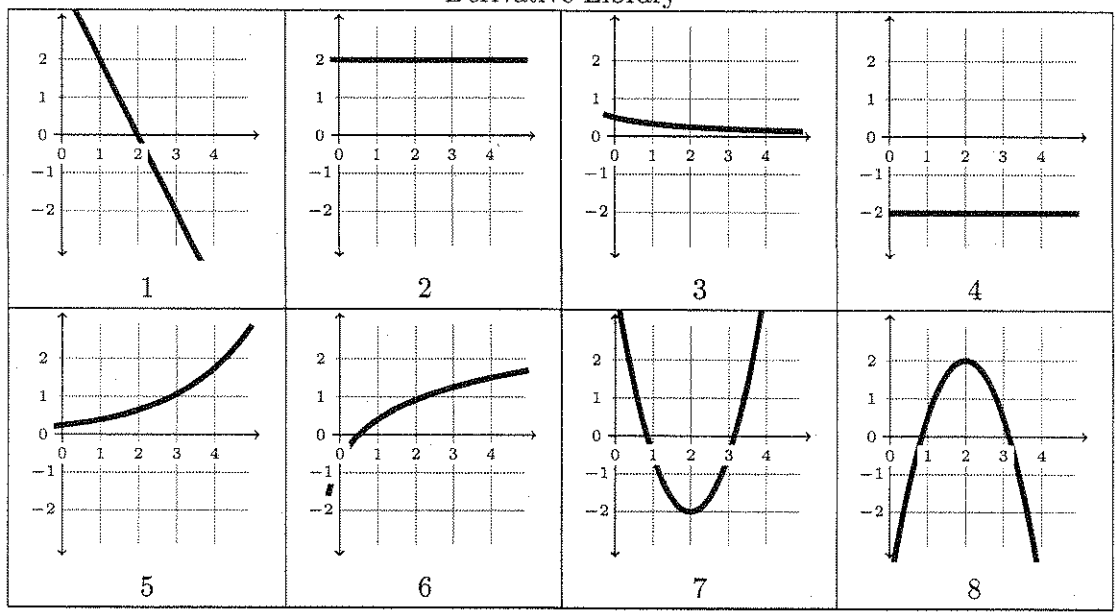
(c)  6

(d)  1

(e)  4

(f)  8

Derivative Library



7. Let $f(x) = -3x^2$.

(a) [**3 points**] Find the average rate of change of f over the interval $[1, 2]$.

$$\frac{f(2)-f(1)}{2-1} = \frac{(-3\cdot 4)-(-3\cdot 1)}{1} = -12-(-3)$$

$$= -12+3 = \boxed{-9}$$

(b) [**8 points**] Find the average rate of change of f over the interval $[x, x+h]$.

$$\frac{f(x+h)-f(x)}{(x+h)-x} = \frac{-3(x+h)^2-(-3x^2)}{h}$$

$$= \frac{-3(x^2+2xh+h^2)+3x^2}{h}$$

$$= \frac{-3x^2-6xh-3h^2+3x^2}{h}$$

$$= \frac{h(-6x-3h)}{h}$$

$$= \boxed{-6x-3h}$$

(c) [**1 point**] Using part (b), find $f'(x)$.

$$\boxed{f'(x) = -6x}$$

Name: _____

Do not turn the page until instructed.

Directions:

1. Write your name on this page and, after the test begins, on the first page of the test.

2. Round all numerical answers to two (2) decimal places.

3. Show your work unless you are instructed otherwise. No credit for answers without work.

4. You may use a calculator provided it is not equipped with a Computer Algebra System (CAS).

5. Turn off and put away all other electronic equipment (especially cell phones), notes, and books.

6. Good luck!