Name: Key

Show your work. Answers without work earn reduced credit.

1. [**3 parts, 1 point each**] Let $C(q)$ represent the cost, $R(q)$ the revenue, and $\pi(q)$ the total profit (in dollars) of producing q units.

   (a) We know that $C'(40) = 83$ and $R'(40) = 50$. Approximate the change in profit if production is increased from 40 units to 41 units.

   $$\Delta\pi \approx \pi'(40) = R'(40) - C'(40) = 50 - 83 = \boxed{-\$33 \text{ dollars}}$$

   (b) We know that $C'(102) = 70$ and $R'(102) = 89$. Approximate the change in profit if production is increased from 102 units to 103 units.

   $$\Delta\pi \approx \pi'(102) = 89 - 70 = \boxed{\$19}$$

   (c) The profit function $\pi(q)$ is maximized when $q = 175$. What is the relationship between $C'(175)$ and $R'(175)$?

   $$MC = MR$$
   $$\boxed{C'(175) = R'(175)}$$

2. [**3 parts, 1 point each**] The cost function is given by $C(q) = 500 + 10q$.

   (a) Find the marginal cost when the production level is 50 units.

   $$MC = C'(q) = \boxed{10 \text{ dollars per unit}}$$

   (b) Find the average cost when the production level is 50 units.

   $$AC = \frac{C(50)}{50} = \frac{500 + 10\cdot 50}{50} = \boxed{\$20 \text{ per unit}}$$

   (c) When the production level is 50 units, what effect will increasing the production have on the average cost? Explain.

   Since $MC < AC$, increasing production will $\boxed{\text{decrease}}$ average cost.

3. **[2 parts, 1 point each]** At a price of \$5 per ticket, a musical theater group can fill every seat in the theater, which has a capacity of 1200. For every additional dollar charged, the number of people buying tickets decreases by 75.

   (a) Find the demand q for tickets in terms of the ticket price p. *[Hint: The demand q is a linear function of p. Once you know the slope and a point on the line, you can use the point-slope formula to write down the equation.]*

· $m = -75$ · $(q_0, p_0) = (1200, 5)$

<u>Eqn:</u>

$$q - q_0 = m(p - p_0)$$

$$q - 1200 = -75(p - 5)$$

$$q - 1200 = -75p + 375$$

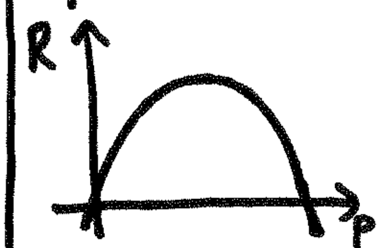
$$\boxed{q = -75p + 1575}$$

   (b) What ticket price maximizes revenue?

· $R = q \cdot p = (-75p + 1575)p$

$$= -75p^2 + 1575p$$

parabola, opens down:



$$R' = -150p + 1575$$

· <u>Set $R' = 0$:</u> $-150p + 1575 = 0$

$$-150p = -1575$$

$$p = 10.5$$

∴ Revenue is maximized when tickets cost $\boxed{\$10.50}$!