Name: Key

Show your work. Answers without work earn reduced credit.

1. [**2 parts, 1 point each**] Let $C(q)$ represent the cost and $R(q)$ represent the revenue, in dollars, of producing q items.

   (a) If $C(50) = 2340$ and $C'(50) = 14$, estimate $C(52)$.

   $$C(52) \approx 2340 + 2 \cdot 14 = \boxed{\$2368}$$

   (b) If $C'(50) = 20$ and $R'(50) = 26$, estimate the profit that the company earns from the 51^{st} item.

   $$\text{Profit} \approx MR - MC = 26 - 20 = \boxed{\$6}$$

2. [**4 parts, 1 point each**] Differentiate the following functions.

   (a) $y = 5x^3$

   $$\frac{d}{dx}[5x^3] = 5\frac{d}{dx}[x^3] = \boxed{15x^2}$$

   (b) $y = \frac{1}{t^4}$

   $$\frac{d}{dt}\left[\frac{1}{t^4}\right] = \frac{d}{dt}[t^{-4}] = -4t^{-5} = \boxed{\frac{-4}{t^5}}$$

   (c) $f(r) = \sqrt{r}(r+1)$

   $$\frac{d}{dr}[\sqrt{r}(r+1)] = \frac{d}{dr}[r^{3/2} + r^{1/2}]$$
   $$= \boxed{\frac{3}{2}r^{\frac{1}{2}} + \frac{1}{2}r^{-\frac{1}{2}} = \frac{3}{2}\sqrt{r} + \frac{1}{2\sqrt{r}}}$$

   (d) $y = x^{\ln 6} + \sqrt{\pi}$

   $$\frac{d}{dx}\left[x^{\ln 6} + \sqrt{\pi}\right]$$
   $$= \boxed{\ln 6 \cdot x^{\ln(6)-1}} + 0$$

3. [**1 point**] Find the equation of the tangent line to the curve $f(t) = t^2 - 3t + 1$ at $t = 2$.

$f'(t) = 2t - 3$

$m = f'(2) = 4 - 3 = 1$

$x_1 = 2$

$y_1 = 2^2 - 3 \cdot 2 + 1 = 4 - 6 + 1 = -1$

$y - y_1 = m(x - x_1)$

$y - (-1) = 1(x - 2)$

$y + 1 = x - 2$

$\boxed{y = x - 3}$

4. [**3 parts, 1 point each**] Differentiate the following functions.

(a) $f(x) = 2e^x + x^2$

$$\frac{d}{dx}\left[2e^x + x^2\right] = \frac{d}{dx}\left[2e^x\right] + \frac{d}{dx}\left[x^2\right]$$

$$= \boxed{2e^x + 2x}$$

(b) $y = 2^t + e^{3t}$

$$\frac{d}{dt}\left[2^t + e^{3t}\right] = \frac{d}{dt}\left[2^t\right] + \frac{d}{dt}\left[e^{3t}\right]$$

$$= \boxed{\ln(2) \cdot 2^t + 3e^{3t}}$$

(c) $g(s) = 4 \cdot e^{0.5s} + \ln(s)$

$$\frac{d}{ds}\left[4 \cdot e^{0.5s} + \ln(s)\right] = \frac{d}{ds}\left[4 \cdot e^{0.5s}\right] + \frac{d}{ds}\left[\ln(s)\right]$$

$$= 4(0.5)e^{0.5s} + \frac{1}{s}$$

$$= \boxed{2e^{0.5s} + \frac{1}{s}}$$